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The implied volatility bias and option smile: is there a simple explanation?

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ABSTRACT

Over the past 30 years, numerous option valuation models have been proposed hoping to explain the volatility smile and the volatility bias shown in the data. However, the Black and Scholes model remains the cornerstone of the option valuation model and its implied volatilities remain essential for calibrating parameters of the other option-valuation models. This research is the first in the literature to use a partial equilibrium model to explain the implied volatility bias using demand for and supply of the options market. The proposed theoretical model allows us to explain the existence of an upward bias and its determinants, and to simultaneously explain both the volatility smile and the volatility term structure. With data spanning the period of 1990 to 2008, twenty-six options on commodity futures markets are analyzed. For at-the-money options, as predicted by the proposed model, the implied volatility is found to be an upward-biased estimator of the realized volatility in nineteen markets. The implied volatility appears to be an unbiased estimator for the realized volatility in the cotton, oats, wheat No. 2, cocoa, orange juice, and heating oil markets. However, for out-of-the-money and in-of-the-money options, implied volatility appears to be an upward-biased estimator of the realized volatility. The theoretical model further suggests that the implied volatility's bias is caused by the quantity hedged, the strike, volatility, futures price, the risk-free rate, option prices, and days to maturity. The bias is different across strikes, times to maturity, puts and calls, option year, and exchanges. In most markets, the open interest and the historical return variables do not appear to have much impact. However, the historical volatility, the Risk free rate, and the Option price variables are shown to have a positive impact on the bias in most markets. The empirical model appears to explain the bias reasonably well with 30%-40% R^2 in eleven markets and more than 50% R^2 in thirteen markets. The results suggest that one should subtract the average bias presented here from the actual option premium before obtaining the implied volatility of the options. This could provide implied volatility which is a more accurate predictor of the future realized volatility.

Keywords: Option smile, Bias in Implied Volatility, Implied Volatility, Options Markets.

CHAPTER 1. INTRODUCTION

In 2000, while working at JPMorgan Chase, David Li became well-known through his paper "On Default Correlation: A Copula Function Approach" published in the Journal of Fixed Income. His paper proposed an elegant formula, known as the Gaussian Copula Function, to value collateralized debt obligations (CDOs). His model has been widely adopted by practitioners due in part to its simplicity. Since its first use, the biggest limitation of this formula, the assumption of constant correlation among assets, has been well known. However, this weakness was apparently ignored by Wall Street traders. At its peak, the Gaussian Copula formula was used to price hundreds of billions of dollars' worth of CDOs. Amid the breakdown associated with the global financial crisis during 2008 and 2009, the incorrect understanding and use of the Gaussian Copula formula is believed to be one of the factors that lead the financial industries into the greatest failure since the Great Depression[1].

In order to develop relevant economic or financial models, researchers often must restrict the model assumptions or assume away the actual complexity of the real world. Model users, on the other hand, should be aware of these limitations. The Gaussian Copula story has shown that, in the highly leveraged world of derivatives, the impact of mispricing is potentially enormous.

The Black-Scholes model (BSM) is similar to the Gaussian Copula formula in the sense that the Black-Scholes model also depends on a set of strong assumptions. BSM assumes constant volatility and frictionless markets. However, unlike users of the Gaussian Copula formula, users of the Black-Scholes model apparently take these limitations into account when employing the model. As will be discussed later the non-constant shape of implied volatility is evidence of the market's correction of the imperfection in the BSM. Despite voluminous research attempting to derive new option valuation models that can better fit the market data, the BSM remains the cornerstone of option valuation due to its speed and simplicity.

Along with the popularity of the BSM, the use of implied volatility has also increased dramatically. According to the BSM, the implied volatility inverted from the option price can be interpreted as the volatility of the underlying asset over the remaining life of the option. Hence, if the BSM is correct, the implied volatility should be the best predictor of future volatility because, by

[1] http://www.wired.com/techbiz/it/magazine/17-03/wp_quant?currentPage=all accessed on February 28, 2009
http://www.lrb.co.uk/v30/n09/mack01_.html, accessed on October 4, 2008

definition, the implied volatility is the future volatility expected by the market. This realization is crucial because future volatility is one of the most important components in asset pricing and risk management used by a wide range of market participants. Option traders rely on future volatility when calculating the probability of future prices falling between certain ranges when constructing trading strategies. Insurance companies rely on future volatility when calculating their insurance premiums. Companies rely on future volatility when performing risk analysis. As exotic options have gained popularity over the past several years, the implied volatility has been used to calibrate inputs used to price these options.

Concerns about the correctness and precision of implied volatility have developed over time. First, according to the BSM, the implied volatility of any option represents the future realized volatility and should be constant regardless of option strikes or times to maturity (Black and Scholes, 1973). Unfortunately, much of the research conducted over the past four decades has shown a non-constant implied volatility in various markets over different time periods. This phenomenon is well-known as the "Volatility Smile[2]" (Rubinstein, 1985, 1994) and "Volatility Term Structure[3]" (Hull 2006). This abnormality of the implied volatility raises questions as to whether the specification of the BSM is correct and whether the assumptions imposed by the BSM are appropriate, particularly those regarding constant volatility and frictionless markets.

Second, the concern about the ability of the implied volatility to predict the future realized volatility has been widely addressed and examined[4]. Most scholars agree that implied volatility is a biased predictor of realized volatility. However, when compared with other candidates for predicting future volatility, such as historical volatility, the results are found to be mixed. Some studies found that implied volatility is an efficient predictor of future realized volatility and that historical volatility contains no additional information not already incorporated in implied volatility

To address the first concern, researchers have developed new option-valuation models allowing for non-constant volatility. Examples include the stochastic volatility model by Heston (1993), the Jump Diffusion model by Bates (1996), and Deterministic Volatility (Dumas et al (1998). Although these models appear to fit the non-constant volatility data, their hedge effectiveness have proven to be no better than the simple BSM (Dumas et al. (1998)).

The second concern regarding the bias-prone and predictive character of the implied volatility has also been extensively examined and documented. Despite the evidence of bias in most markets,

[2] The non-constant shapes include the shapes of smile, sneer, smirk, and skew over options' moneyness. For the remainder of the paper, we refer to these non-constant shapes as the "Volatility Smile".
[3] Similar to volatility smile, volatility term structure refers to non-constant shapes of the volatility over options' time to maturity.
[4] Chapter 2 provides lists of literature related to this topic.

practitioners continue to use the implied volatility as one candidate for predicting future realized volatility.

1.1 Problem Statement

This research addresses the widely-studied implied volatility bias of the BSM. A voluminous body of literature has attempted to explain the volatility smile through various option-pricing models. I show that when one allows for the demand and supply of the services of option writers, market participants will agree on an equilibrium bias. This bias is nothing more than a fair return to the skills needed and costs associated with option writing. The existence of the bias that is predicted here explains both the observed smile and the volatility term structure. To the best of my knowledge, no existing research examines the implied volatility bias within such a partial equilibrium model. The ability of the model to explain the cause and the size of the implied volatility bias should provide a better estimator of future realized volatility.

1.2 Motivation and Scope

The research is motivated by the fact that there is an increasing usage of implied volatility as an estimator of the future realized volatility. In most cases, researchers have found evidence that the implied volatility is a biased estimator of future realized volatility, but for the most part this evidence has been simply ignored. For example, insurance providers use implied volatility as a proxy for future realized volatility when generating the price distribution to obtain a fair premium. Similarly, option traders use implied volatility as a predictor of future volatility in order to price exotic options. Hence, if implied volatility is actually a biased estimator of future realized volatility, these options and insurance premiums will also be biased. The upward bias in implied volatility will cause these prices to be upwardly biased as well.

To analyze bias in implied volatility, this research attempts to address the following questions. First, does bias in the implied volatility exist and is the actual volatility an upward estimate of expected future volatility? Although most research has found evidence of bias, none have concluded that implied volatility is an upward bias estimator of the realized volatility.

Second, what causes bias in the implied volatility? To answer this question, I propose a partial equilibrium framework in which the BSM option premium is an input. This model makes it possible to explain the bias. Moreover, the model is also designed to allow for bias at different strikes which result in non-constant volatilities across strikes.

Third, does the partial equilibrium model really work? This question is tested empirically. The empirical research examines whether variables suggested by the theoretical model can explain the bias. The result from the empirical study should yield ideas about the size and the direction of the bias. If we discover that the bias can be systematically explained by a set of variables, practitioners could account for these variables when using the implied volatility as an input in for their models. Therefore, in order to obtain a more precise estimator of the future volatility, the implied volatility should be treated appropriately by eliminating the bias components.

1.3 Contribution of the research

The contribution of this research is that it provides the ability to explain economically the sources of the bias from the BSM through interaction between the demand and supply of the services of option writers in the market. Furthermore, I propose an econometric model that can be used to explain the sources of bias. This model allows market participants to better estimate future realized volatility, hence, better manage their risks.

1.4 Summary of subsequent chapters

This paper is organized as follow: Chapter 2) discusses literature review & motivation. Chapter 3) provides the model derivation, and Chapter 4) shows the empirical evidence of bias and the model results. The conclusion regarding further research is then presented in Chapter 5).

CHAPTER 2. LITERATURE REVIEW

This chapter discusses existing literature and provides motivation for the proposed model. Section 2.1 summarizes abnormalities resulting from the Black-Scholes model and reviews alternative option valuation models developed to overcome weaknesses of the Black-Scholes model. Section 2.1 concludes with a discussion of the limitation of these alternative models. Section 2.2 describes the two most tested hypotheses regarding bias and the information contents of the implied volatility. This section also points out the limitation of the current empirical method and proposes an alternative empirical method. Section 2.3 discusses mechanism in the options markets. The discussion in this section provides motivation of the partial equilibrium model proposed here. Section 2.4 provides details regarding this partial equilibrium model. The proposed model bridges the gap in existing literature by using the demand and supply for the services of option writers to explain the existence and determinants of the implied volatility bias.

2.1 The Option pricing literature

Trading actively in many major exchanges throughout the world, options have become increasingly used for hedging and speculation. The collapse of the global financial sector in 2008 has proven the importance of the derivatives such as options in the modern financial system.

In the option world, the Black-Scholes (or Black-Scholes-Merton, or BSM) formula, developed in 1973, is the cornerstone of option pricing. Thanks to its elegance and simplicity, the BSM formula and its variations have been widely adopted among practitioners. However, the speed and simplicity offered by this formula do not come without costs. According to the BSM formula, the volatility input is assumed to be constant across strikes and times to maturity. However, in reality, the volatility derived from the model by equating option market prices to the BSM formula, known as "implied volatility", appears to be anything but constant. The non-constant feature of the implied volatility has gained attention among scholars since it was first discovered. Over time, researchers have discovered that the patterns of volatility differ among markets and appear to change over time. For example, foreign currency options exhibit a completely symmetric "smile" shape over the strike price (Hull, 2006). The volatility smile also appears to have a downward sloping shape for post 1987

S&P 500 futures options[5] (e.g. Hull (2006), Rubinstein (1994), Dumas et al. (1998), Ederington and Guan (2002)) and an upward sloping shape for corn futures options (Ferris et al. (2003)). However, in some markets, the implied volatility does not appear to have a perfect smile shape, even though the phenomenon of all non-constant volatilities is still referred to as the “Volatility Smile”.

Most studies have shown that the implied volatility is a biased estimator of the future volatility (for example, Edey and Elliott (1992), Day and Lewis (1992), Lamoureux and Lastrapes (1993), Jorion (1995), Fleming (1998) and Simon (2002)). However, as described in Section 2.2, with the existing methodology, most research can only conclude whether the implied volatility is a biased estimator of the future volatility or whether the implied volatility is more or less volatile than the future volatility. In most cases, the definite conclusion about the direction of the bias cannot be drawn. Although most studies agree on the biasness of the implied volatility, a handful of research studies disagree that the implied volatility is a biased estimator of the realized volatility. For example, consider Christensen and Prabhala (1998), who argue that the difference in their results could contribute to a longer time span using non-overlapping sampling method[6]. They also suggest that the shift in the October, 1987 crash might explain the bias found in other research.

The discovery of the volatility smile and the bias of implied volatility have raised questions among researchers about the validity of the BSM. Several studies have examined whether sample variation, measurement errors[7], or sample selection bias[8] could cause these abnormalities. Measurement errors from asynchronous prices and bid-ask spreads in options and futures could also be a factor that causes bias in the implied volatility. However, several researchers such as Jorion (1995) consider 30 basis points bid-ask spread for options on foreign exchange that have no effect on the estimates of the implied volatility bias. Therefore, with the volatility smile observed over varieties of markets and periods of times, these explanations are too weak to explain the cause of the smile.

Another possibility for explaining the volatility smile is the specification error of the BSM. Since the BSM is based on several strong assumptions, violation of these assumptions could lead to results that are not consistent with the model's prediction. The price of volatility risk and the fat tail

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[5] Over the range of strike values, the implied volatility of the S&P 500 index has maintained the sneer shape since the stock market crash in 1987. This effect corresponds to the common belief that bearish markets are more risky than bullish markets. That is because in bearish markets companies tend to increase their leverage because issuing equity is more difficult. As a result, companies become more risky and their implied volatility increases. Prior to the crash, the implied volatility for the S&P 500 index was much less depending on strike price (Hull 2006).

[6] Overlapping sample methodology tends to yield less precise and potentially inconsistent estimates (Christensen & Prabhala (1995)).

[7] Numerous research studies have been dedicated to choosing the best weighting scheme to estimate implied volatility. However, Poteshman (2000) suggests that sampling variation should not cause the bias. In addition, Neely (2004) points out that sampling variation can be eliminated using a long span of data such as 12 years. Bates (2000) also provides evidence that the implied volatility is not sensitive to the option pricing model.

[8]In later work by Flemming (1998), he suggests that the bias might come from the non-zero correlation between price and volatility and the American style options (as compared to the European style options calculated by the Black-Scholes model)

distribution are among the factors most discussed in the literature. Lamoureux and Lastrapes (1993) agree that a price of volatility risk is likely to be responsible for bias in the implied volatility for the case of options on stocks. Doran et al. (2005) also indicate that the bias of implied volatility results not only from the price of volatility risk, but also from the presence of stochastic volatility and jumps. That is, the actual price distributions have fatter tails than that of the normal distribution assumed by the BSM.

In addition to the volatility risk and the unrealistic normal distribution assumption, the BSM also relies on another restricted assumption, namely, a frictionless market. In reality, transaction costs, market liquidity, and other elements of market friction are inevitable components of the markets. Market frictions create risks and prevent the market from completion[9]. The violation of the frictionless market assumption could lead to a serious issue with respect to model validity. Boyle and Vorst (1992) and Leland (1985) and Longstaff (1995) argue that, with the presence of transaction costs and other frictional elements, the value of a replicating portfolio of an option must be discounted using path dependent probabilities and that option prices need not follow the martingale condition. Hence, the options must be priced using an equilibrium model rather than using the no arbitrage model. Although researchers are still engaged in trying to explain the BSM bias[10], this hasn't slowed down its usage among practitioners. The model is widely used to price vanilla options. Moreover, since the implied volatility, by definition, represents future realized volatility over the life of the options, it is often used to estimate future realized volatility (or realized volatility). As derivatives markets become increasingly important, the use of implied volatility has expanded. Implied volatility is used to calibrate input parameters when pricing exotic options and insurance premiums. It is also used as an input for dynamic hedging strategies and risk analysis.

As the market has become more and more reliant upon speed and simplicity, the BSM has become widely adopted in the derivatives community. The volatility smile and the bias from implied volatility has also become widely accepted as a result of the model. However, the volatility smile creates a more serious problem, because it represents an internal inconsistency between the model's assumptions and reality. This is because the volatility smile violates the constant-volatility assumption of the BSM that is required before the implied volatility can be generated. The volatility abnormalities cannot be just simple mispricing of the options, because if so, arbitragers would step in to buy cheap options and sell more expensive options. Over time, the prices of options would converge to their actual prices. However, this is not the case. The volatility smile and volatility bias

[9] That is, the market is incomplete.
[10] Mayhew (1995) also provides excellent earlier survey of empirical results testing the Black and Scholes model.

persist in different markets over different time periods. Hence, any price deviation indicates that it is the level that the market agrees upon. As a result, research questions such as: what causes this volatility smile and volatility bias? and how should we develop the option-valuation models that can better fit the data have become the focal point for hundreds of research studies over the past several decades.

The second part of the questions regarding searching for new models that can be used to explain abnormalities in implied volatility will be discussed later. The first part of the question regarding the explanation of the smile and bias will be discussed as follow.

To understand how researchers develop new models in hopes of correcting the shortfalls of the BSM, it is important to understand how the BSM is derived. The derivation of the BSM is both simple and elegant and can be done through the non-arbitrage condition or through the Risk-Neutral Valuation Relation (RNVR). Through the non-arbitrage condition, the number of options hedged against the underlying asset depends on the strike price, the current price of the underlying asset, the time to option expiration, the interest rate on a risk-free bond, and the future stochastic process for the underlying asset price. When these factors are known, the proper hedging portfolio is known and an equilibrium price can be found that is invariant to risk preferences and to expected changes in the underlying commodity price.

If the BSM is derived through the RNVR technique, it can be determined either with or without preference assumptions. Through the non-preference assumption, the RNVR can be determined by converting the physical probability into the risk-neutral probability. Hence, the option payoff can be discounted using the risk-free rate (Cox, Ross and Rubinstein (1979))[11]. The preference-based RNVR relies on a general equilibrium model. By assuming different combinations of utility functions and underlying asset price processes, and by solving a representative agent's utility maximization problem, the RNVR can be recovered, leading to the original BSM formula. Several studies have found that the RNVR can be derived through Constant Relative Risk Aversion (CRRA) preferences and a lognormal distribution of underlying asset price (Merton (1973)), CRRA preferences and a bivariate lognormal distribution of the return on the underlying asset and the return on aggregate wealth (Rubinstein (1976)), Constant Absolute Risk Aversion (CARA) and a bivariate normal distribution of the price of the underlying asset and aggregate wealth (Brennan (1979)), and exponential risk preferences and a transformed normal distribution of aggregate wealth and the underlying asset price (Câmara (2003)). The most recent development in this field is to use Epstein-

[11] Sundaram (1997) provides an excellent intuitive explanation about how the RNVR works and how it is equivalent to the Black and Scholes model.

Zin preferences and multivariate affine jump diffusion underlying state variables (Eraker & Shaliastovich (2007)).

These new models typically alter the original the BSM assumptions or develop new techniques[12] for determining price options. For example, the univariate diffusion model relaxes the geometric Brownian motion assumption (e.g., Cox and Ross (1976), Cox and Rubinstein (1985)[13]), the stochastic volatility and jump process models relax the constant-volatility assumption and the underlying price process assumption, respectively (e.g. Hull & White (1987) and Heston (1993)), the deterministic volatility model allows the volatility to be locally deterministic (Derman and Kani (1994a,b), Dupire (1994), and Rubinstein (1994)) and, finally, discrete time models such as the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models relax the volatility process structure (Bollerslev (1986), Duan(1995) and Heston and Nandi (2000))[14]. Several extensions allowing for trading costs, short-sales constraints, and other market friction elements have also been developed (for example, Leland (1985), Hodges and Neuberger (1989), Bensaid et al. (1992), Boyle and Vorst (1992), Karatzas and Kou (1996), and Broadie et al. (1998)[15]). Bakshi et al. (1997) list more studies analyzing these new option-pricing models.

By relaxing assumptions regarding the volatility process, these models can successfully explain the volatility smile. For example, Dumas et al. (1998) refers to the work by Taylor and Xu (1993) wherein they demonstrate that more complex valuation models such as jump diffusion can generate time-dependence in the sneer even when volatility is constant over time. Dumas (1998) also gives an example of stochastic volatility by Heston (1993) and Hull and White (1987) that can explain the sneer when the asset price and volatility are negatively correlated, because negative correlation helps create the sneer. They also refer to the jump model of Bates (1996a) that is able to create the sneer if the mean of the jump is negative. However, the predicting and hedging performance of these models are still questionable. For example, Dumas et al. (2000) find that the deterministic volatility function of option price performed worse than the ad-hoc Black and Scholes model in terms of hedging and predicting out-of-sample option values.

However, instead of replacing the BSM models in the market place, a growing number of these sophisticated models makes the BSM even more important. This is because the increase in

[12] The most recent technique used in the new developed model is the Fourier inversion approach used in Stein and Stein (1991), Heston (1993) obtained these studies from Bates (2003)) and Eraker and Shaliastovich (2007).
[13] These studies are listed in Bates (2003)
[14] Christoffersen and Jacobs (2004) provide a number of references using the GARCH model.
[15] These studies are listed in Broadie and Detemple (2004)

option complexity leads to an increasing usage of Monte Carlo techniques in which the model parameters need to be calibrated using market data. Market data are typically collected from options traded on the exchanges. These options are mostly plain vanilla and are calculated using the BSM because of its speed and simplicity. Therefore, the implied volatility inverted from the BSM has become necessary data for these newly developed models.

Not only do the data from the BSM become necessary input for these models, but also the models themselves have a major drawback, a lack of speed. To price plain vanilla options, these models rely on a Monte Carlo technique, a major obstruction for practitioners. Monte Carlo methods require a relatively longer time to price the options and also require time intensive calculations. By the time the models produce fair option prices, market conditions will have changed and the prices will no longer represent the current market environment. Compared with more sophisticated models, the BSM can produce prices very quickly. Although, in many cases, these prices are only quick and dirty estimates, for options traders who understand the market, these prices are usually adequate to use as a first step in making the market.

When practitioners choose to use the more sophisticated models, the immediate question that should be considered is whether the benefits outweigh the costs. Bakshi et al. (1997) ask whether we gain anything from more complicated models and whether they are able to correct for the biases associated with the BSM. Their question is crucial for this research. As we examine the existence of bias in various commodity markets and progress in the search for the best model to explain the bias embedded in the implied volatility, we must keep in mind that the chosen model should be simple enough to be understood by practitioners and should reflect the reality of the market.

2.2 Biasness and Informational efficiency hypotheses

After almost four decades and at least three major option valuation models, the BSM is still the cornerstone for the options market. This brings us back to the first question posted in the previous section, i.e., what causes the volatility smile and volatility bias? As the market continues to use the BSM as a valuation model and implied volatility as an estimate of future realized volatility, the sources of the volatility smile and volatility bias have become especially important.

In the literature, the conditional bias tendency of the implied volatility is generally tested using the following hypothesis:

$$\sigma^2_{RV,t,T} = \alpha + \beta\sigma^2_{IV,t,T} + \varepsilon_t \qquad (2.1)$$

where $\sigma^2_{RV,t,T}$ and $\sigma^2_{IV,t,T}$ are the subsequently realized volatility and the implied volatility between period *t* and *T* respectively. The rejection of the null hypothesis that α=0 and β=1 indicates that the implied volatility is a biased estimator of the realized volatility.

To test whether the implied volatility is informationally efficient, the following regression is typically conducted:

$$\sigma^2_{RV,t,T} = \alpha + \beta\sigma^2_{IV,t,T} + \gamma\sigma^2_{FV,t,T} + \varepsilon_t \tag{2.2}$$

where $\sigma^2_{FV,t,T}$ is an alternative forecast of volatility from period *t* to *T*. The rejection of the null hypothesis $\gamma = 0$ would lead to the rejection of the notion that the implied volatility is informationally efficient in predicting the realized volatility.

Testing across asset classes over different periods of time, researchers have found $\hat{\alpha}$ to be positive and $\hat{\beta}$ to be less than 1. Most researchers agree that the implied volatility is a conditionally biased estimator of the subsequently realized volatility (Neely, 2004). However, setting up the empirical model this way does not directly allow researchers to conclude whether the implied volatility *overestimates or underestimates* the realized volatility. This is because the positive values of the $\hat{\alpha}$ and $\hat{\beta}$ do not guarantee that the $\hat{\sigma}^2_{IV,t,T}$ will be an upward estimator of $\sigma^2_{RV,t,T}$. As a result, only a few research studies have been able to make a conclusion about the direction of the bias of the implied volatility. For example, Bates (2003) shows $\hat{\alpha}$ to be 0.0027 and $\hat{\beta}$ to be 0.681. From this result, he concludes that the implied volatility overestimates the realized volatility due to a small intercept and a significantly greater than zero slope. For most research studies, the estimates do not allow drawing of a conclusive direction about the implied volatility. For example, Jorion (1995) tests similar hypotheses using non-over lapping data for three foreign exchange markets: The German deutsche mark, Japanese yen, and Swiss franc during the period 1985 to 1992. He finds that $\hat{\alpha}$ is approximately 0.3 and $\hat{\beta}$ is approximately 0.5. Based on these estimates, the only conclusion that can be drawn is that the implied volatility is more volatile than the realized volatility; that is, the implied volatility should be scaled down when relatively higher than average and scaled up when relatively lower than average. No conclusion about the direction of the bias is provided.

Using the non-overlapping data of the S&P 100 index between 1986 and 2004, the S&P 500 index between 1994 and 2004, and natural gas, crude oil, and heating oil prices between 1995 and 2005, Doran et al. (2005) employ the instrumental variable technique to construct the estimated implied volatility and use the estimated implied volatility to estimate equation (2.2). They conclude that since the slope is less than one and the intercept is either positive or negative, the implied volatility is an upward biased estimator of the realized volatility. They attribute the implied volatility bias to the negative market price of volatility risk. However, I disagree with the approach they use to draw a conclusion because, as pointed out earlier, the positive slope, even when combined with a positive intercept, does not guarantee that the implied volatility will be an upward bias estimator of the realized volatility.

The conclusion that the implied volatility overestimates the realized volatility is only true under certain situation, such as when the slope is less than one and the positive intercept is small. Due to this restriction, the empirical analysis is designed to account for this issue by constructing the dependent variable as the difference between the implied volatility and the realized volatility. Hence, the conclusion about the bias of the implied volatility can be drawn. When the intercept is negative (positive), the implied volatility overestimates (underestimates) the realized volatility.

Since most researchers agree that the implied volatility is a biased estimator of the realized volatility, the next mission is to determine the cause of this bias. Several hypotheses have been tested. Neely (2004) tests whether different kinds of measurement error can cause the bias. These measurement errors include error from using high-frequency options data, error from using horizon-by-horizon estimation, error from sample selection bias, and error from price of volatility risk. In the end, he finds no explanation of the bias through any measurement errors. Different conclusions were found by Doran et al. (2005), who examined the bias and concluded that it is a result of the market price of volatility risk. Other researchers (e.g. Longstaff (1995), Dennis and Mayhew (2001) and Aijo (2002)) examine the bias through the demand and supply framework, but neither conclusions nor theoretical models offer. This research will bridge the gap in the literature. The model proposed in Chapter 3 not only provides testable hypotheses, but also satisfies economic intuition about the causes of the bias.

The information content of the implied volatility is another aspect of the implied volatility that is being extensively examined by researchers. Once again, they are unable to find a common ground regarding the information content embedded in the implied volatility. This is partly due to different sampling and measuring techniques, different estimation methodologies, and different markets. Using a simple BSM without accounting for dividend or early exercise rights, earlier researchers found that the implied volatility from stock options can explain the realized volatility better than can the historical volatility (Szakmary et al. 2003). However, in 1988, Christensen and Prabhala (1988) using monthly observations, constructed non-overlapping data and found that the implied volatility is a good predictor of the realized volatility. With the development of more sophisticated sampling and testing methods, results from more recent studies have been mixed (Szakmary et al. 2003). Canina and Figlewski (1993) examine options on the S&P 100 index using a regression approach and find that there is no relation between implied volatility and realized volatility. Day and Lewis (1992) study the at-the-money options on the S&P 100 index and find that, although the implied volatility does contain some predictive power, time series models such as GARCH or the historical volatility do help improve this predictive power.

Szakmary et al. (2003) analyzed options from 35 futures markets traded over eight exchanges and found that implied volatility is a good predictor of the realized volatility and that the time series model such as the moving average and the GARCH model contain no predictive information that is not embedded in the implied volatility. They conclude that the futures options markets are efficient.

In more recent work, Neely (2004) attempts to derive the explanation for the bias and the inefficiency of implied volatility by estimating by means of the stochastic volatility model. He corrects for the overlapping data (telescoping samples) by constructing an appropriate covariance estimator following Jorion (1995) using horizon-by-horizon estimation. However, he still finds that implied volatility is a biased estimator for realized volatility and that the foreign exchange market is inefficient. His paper also rejects the hypothesis that the non-zero price of volatility risk generates the bias in the implied volatility.

Despite voluminous empirical works conducted over the past 30 years, it is still debatable whether implied volatility is a biased estimator of realized volatility and whether the information content embedded in the implied volatility is efficient enough to predict future realized volatility.

2.3 What do we need to know about the options market?

It has been almost four decades since the BSM was introduced and researchers are still struggling trying to pinpoint the explanation of the volatility smiles and volatility bias expressed in the data. The focuses of previous research have been either trying to derive an option valuation model through new variants of stochastic processes of the underlying assets or testing the bias and information content of the implied volatility. Yet, no final conclusion has been reached. This research proposes to explain the abnormalities of the implied volatility using a different approach - the partial equilibrium approach. I focus on the fundamental factors of the options market, the interaction between demand for and supply of options, and the structure and mechanism of the options market.

To understand how the demand for and supply of options function, we must first understand how options are traded. Option transactions are similar to ordinary stock market transactions where market makers (or traders) quote bid-ask prices and help in facilitating the price discovery process. Therefore, the following observations about their behavior should help us understand how the options market operates and what, where, and how the BSM fails to capture important information.

First, according to an interview with option traders from the heating oil pit at the New York Mercantile Exchange (NYMEX) in May 2007, option traders admit that, though a numbers of model can be used to price options, they use BSM[16] due to its speed and simplicity. Prices calculated from the BSM are considered to be "good enough" because traders do not try to quote the most accurate option prices. Instead, the ability to price options satisfactorily relative to their peers is more important than the ability to price the options accurately. In fact, some traders have not changed the interest rate input of the BSM in months[17]! Most traders would agree that they do not know much about the real values of the options until they are approaching maturity. This observation is also true for electronic trading.

Second, most option traders are neither statisticians nor mathematicians[18]. They do not understand complicated option valuation models. Though most traders on the exchanges use handheld computers (so-called, "the Tablet") to calculate option prices, they find the simplicity of the BSM (or its slight variations) most attractive to them because the BSM only leaves the "volatility" as a subjective input. As a result, traders can price any option by simply changing one parameter in the pricing model, the volatility.

Third, since the volatility is the only unknown variable needed in their calculation, option writers can incorporate their market perspective into option prices. At the beginning of each trading day, the starting volatility parameter is the implied volatility inverted from the previous day's option price. Throughout the day, as market conditions change, e.g., if more demand (supply) enters the market, traders respond to demand changes by increasing (decreasing) option prices. This can be done by increasing (decreasing) the volatility parameter. Hence, whether traders truly interpret the volatility parameter as an estimation of realized volatility over the remaining life of an option is questionable. However, from the option traders' perspective, volatility is clearly used as a link

[16] Note that, these option traders can also be considered as market makers for the option markets in which they participate.
[17] For short dated options, the impact of interest rate is very small and, hence, mostly ignored by traders.
[18] Note that we are focusing on option traders who act as market makers. The designations option traders, traders, or option writers are used interchangeably throughout this research.

between the option price, demand and supply of the option market, and traders' perception of the market to reflect the fair value of the options.

Fourth, trading options incurs costs. Trading costs include cost of exchange seats, opportunity cost of skills, time, and hedging. Option traders possess skills and economies of scale to dynamically manage their risks. In the competitive market, traders' skills can be used elsewhere unless they are compensated for their costs and skills.

Fifth, unlike other businesses, the option market does not directly create wealth in the economy. Instead, it provides a risk-shifting service that allows businesses to focus on their core business activities that can help increase wealth (Baird (1993)). In a sense, option writers act like an insurance company to bear extra risks for which they should be compensated.

Finally, sixth, the market generally believes that the probability of extreme events in the real world is higher than the normal distribution assumed in the BSM. In other words, the real world distribution appears to have a fatter tail than the normal distribution.

These observations clearly violate the BSM assumption regarding constant volatility and frictionless markets. Therefore, the non-arbitrage condition and the RNVR need not to be satisfied and the option valuation model is no longer valid, so the options should be priced through the equilibrium model (Longstaff (1995)). Therefore, the question that needs to be asked is: if option markets and option traders behave as I described earlier, should option prices quoted by these traders reflect their behavior? If so, how do we capture this behavior?

2.4 A Partial Equilibrium Model – An Explanation for the Implied Volatility Bias

The previous discussion about how the option market operates sheds some light on how to explain the bias embedded in option prices. The key point to remember is that, regardless of any weaknesses resulting from the BSM, it is still widely used among practitioners. To compensate for the model's unrealistic assumptions, the market instead imposes an adjustment to prices derived from the model. The only input parameter that allows market to incorporate this adjustment is the volatility input; hence, the result of volatility smiles or volatility biases is seen in the data.

In contrast to previous studies whose primary objective is to search for a perfect valuation model or to test for the degree of bias and information content of the implied volatility, the focus of this study is to model the imperfection of the current valuation model. That is, I attempt to explain the bias embedded in option prices using the partial equilibrium of the interaction between supply and demand for the services of option writers rather than to develop a new valuation model.

The above observations suggest that the market price of the option is comprised of two different components, the theoretical value from the BSM and the bias adjustment. When options are transacted, buyers and sellers of options must agree on the market price, hence the theoretical values and the bias adjustments. If the theoretical values are known, the problem reduces to finding the equilibrium price of the bias. In equilibrium, demand and supply of the bias must be equal. Intuitively, bias supply comes from costs and additional risks that option traders must bear, and bias demand comes from option buyers who are willing to pay a price premium in order to reduce their risks and to compensate skilled traders to rebalance their risks rather than doing it themselves.

The idea of explaining volatility smiles and volatility biases through the demand and supply framework has been mentioned on several occasions in the literature. However, it has received almost no attention and has never been modeled systematically because most attention is directed toward creating new valuation models that can explain volatility bias and volatility smiles in the data. These new models do not take into account the fact that the BSM is still the model used by traders.

In papers that are closest to this research, Fleming (1998) finds an upward bias in S&P 100 futures options. He goes on and suggests that a linear model that corrects for the implied volatility bias can provide a useful market-based estimator of conditional volatility, in other words, the implied volatility bias can be modeled as a function of a set of explanatory variables. Dumas, Fleming and Whaley (1998) use an ad-hoc version of the BSM which specifies that the BSM implied volatility is a linear function of the strike price, strike price squared, time to maturity, and time to maturity squared. Although these studies do not provide a theoretical explanation of their models, their methodology is consistent with the assumption that we can view the bias in the implied volatility as a function of sets of explanatory variables resulting from the demand and supply of options. Bates (2000) indicates that option market practitioners believe that heavy demand for out-of-the-money (OTM) put options has driven up their prices. This could be one of the possible explanations of the implied volatility smirk after the 1987 crash[19]. However, in his paper, Bates chooses to explain this abnormality using different stochastic volatility and jump-diffusion models instead of using a demand-supply perspective.

Some recent research on the option bias is directed toward using a demand and supply framework. For example, Ferreira, Gaga, Leon, and Rubio (2005) provide a discussion on the impact

[19] Since the 1987 stock market crash, the implied volatility of S&P 500 futures options are known to have the volatility smirk pattern (Bates (2000)).

of net buying pressure and limited supply. Bates (2003), Whaley (2003), and Bollen and Whaley (2004) discuss the importance of a net buying effect on the option bias. However, this research takes a different approach in which the bias is determined from a partial equilibrium framework that contains broader economic insight. Equilibrium results from an agent's expected utility maximization problem. Because of the nature of options trading, options writers must purchase or lease exchange seats in order to become market makers. If they do not own a seat then option writers must pay service fees to have their contracts traded. The need to own a seat and the skills required to write options are barriers to entry in for option writer. Hence, I introduce monopoly power into the market.

CHAPTER 3. MODEL

The model presented in this chapter is different than those presented by Brennan (1979) and Rubinstein (1976) that are based on a general equilibrium setting. However, for completeness, I present Brennan's model to show how asset prices are derived in this general equilibrium setting.

Brennan (1979) extends the work by Merton (1973) and Rubinstein (1976) to show that the BSM can be derived by assuming Constant Absolute Risk Aversion (CARA) preferences combined with the normality assumption of the price of the underlying asset. The following section briefly describes Brennan's model. His methodology and assumptions will be applied to my model later. In his model, Brennan assumes separability in a two-period utility function and period-zero endowment. Hence, a representative investor faces the following expected utility maximization problem:

$$Max_{\{C_0, X_j\}} \quad U(C_0) + EV[W_1] \tag{3.1}$$

Where

$$W_1 = \left(W_0 - C_0 - \sum_{j=1}^{N} x_j P_{0j} \right) (1 + r_f) + \sum_{j=1}^{N} x_j P_{1j}$$

$$= (W_0 - C_0)(1 + r_f) + \sum_{j=1}^{N} x_j \left(P_{1j} - P_{0j}(1 + r_f) \right)$$

where

$U(.)$	Utility function at period 0,
$V(.)$	Utility function at period 1,
W_0	Initial wealth of a representative investor,
W_1	Period 1 wealth of a representative investor,
C_0	Initial consumption of a representative investor,
x_j	Numbers of units of risky security j purchased,
P_{0j}	Initial price of risky security j (j=1,2,......,n),
P_{1j}	End of period price of risky security j,

r_f Risk-free interest rate,

Solving the optimization problem using the Lagrangian method, First Order Conditions (FOCs) become

$$U'(C_0) - (1 + r_f)E[V'(W_1)] = 0 \tag{3.2}$$

$$E[V'(W_1)P_{ij}] - P_{0j}(1 + r_f)E[V'(W_1)] = 0 \tag{3.3}$$

for *j* = 1,2,.....*N*. Using these FOCs and market clearing conditions are sufficient to determine the P_{0j}. Dropping subscript *j* and rearranging the above equation, we obtain

$$P_0 = \frac{E[V'(W_1)P_1]}{(1 + r_f)E[V'(W_1)]} = \frac{1}{(1 + r_f)}E\left[\frac{V'(W_1)P_1}{E[V'(W_1)]}\right] \tag{3.4}$$

This relationship is true for all assets. Assuming use of the exponential utility function and bivariate normal distribution[20] of the price of the underlying asset and aggregate wealth, this model results in a risk-neutral valuation relation (RNVR) and hence, the BSM formula.

Taking into account common practices in the market, the proposed partial equilibrium model developed here uses results from Brennan and Rubinstein's models, uses the BSM, as an input and adds another layer of equilibrium analysis to account for the bias. This model essentially replicates the market pricing mechanism where the market is adjusting the option price from the BSM. However, from a practical perspective, this adjustment can only be done through volatility, the only unknown input to the model. Note that, in the absence of market frictions, RNVR will hold and the BSM will be valid. In that case, this partial equilibrium model will be unnecessary and the model will reduce to Brennan's model.

My model assumes heterogeneous agents interact in the monopoly market setting. Two cases are considered: 1) no discrimination, and 2) perfect discrimination. No discrimination refers to the case where a monopolist doesn't have the power to charge different prices to different customers. Perfect discrimination is the case where a monopolist can charge different prices to different customers.

The model considers a two-period economy with periods *t = 0, 1*. There are two different types of agents: asset owners and options writer. There are several asset owners but only one option

[20] A normality assumption can be appropriate when an underlying asset can take on negative values (Brennan (1979)).

writer[21]. Both the asset owners and the option writer attempt to maximize their own total utility, which is the sum of utilities at periods 0 and 1. The asset owners are risk-averse. Their utilities at each period are concave and time-separable. The option writer is risk-neutral and writes options in a monopoly market environment. At period 0, both the asset owners and the option writer are equally endowed with physical goods. However, asset owners expect additional physical goods in period 1 (e.g., from harvesting their crops). The price of physical goods at time 0 is known for certain. Although the price at time 1 is uncertain and unknown, parameters that govern the price distribution are public information and therefore known for certain at period 0. Due to price uncertainty, each asset owner decides to buy put options to hedge the future price uncertainty risk. I assume that each asset owner has a unique preference for a unique strike, e.g., each asset owner wants to purchase the options for a particular strike and does not want to purchase any options for any other strikes. Each option provides protection for one unit of physical good. Options trading takes place at time 0. In this model, market participants are assumed to use a Black-Scholes option-pricing model to price the options. With the well-known limitation of the Black-Scholes model, the market imposes an additional price adjustment (the "Bias") to the theoretical price of the options". In equilibrium, options buyers and seller agree on an optimal bias level.

Notation

S	Bias (in dollars)
P_{opt}	The option price calculated from BSM ($P_{opt}+S$ is the real option price that asset owners actually pay)
r_f	Risk free rate of return
a	Risk aversion coefficient (Arrow-Pratt's coefficient of absolute risk aversion)
P_1	Price of underlying asset at time 1
k_i	Strike price preferred by asset owner *i*
Q_i	Quantity of physical goods that asset owner *i* is endowed with in period 1
A_i	Quantity hedged by asset owner *i*
W_{i0}	Initial wealth of asset owner *i*
H	Fixed trading cost
c	Variable trading cost (per Unit of physical good)

[21] This is comparable with the real-world situation. For example, an insurance company sells insurance to several policy holders or there are many options buyers with a limited number of options traders.

3.1 The asset owner's problem

The model assumes *n* types of asset owners with identical and separable utility functions. Each type of asset owner prefers a different strike and has an identical quantity of physical good at period 0. This quantity is known for certain and is also publicly known. The price of the endowment at period 1 is unknown and is assumed to follow a lognormal distribution. There are two types of assets: the endowment and the options on the endowment. Asset owners can purchase put options to hedge against the price uncertainty of the goods during period 1. Asset owners are not allowed to borrow money to buy options.

An asset owner's objective is to allocate his endowment into consumption over two periods to maximize his total utility. An asset owners' utility is assumed to be represented by an exponential utility function. In period 0, each asset owner has to decide the option quantity that he wants to buy to hedge against the price uncertainty. The only difference among asset owner types is that they prefer different strike prices. For example, asset owners of type *i* prefer strike K_i , i.e. their utility can be represented as $U_i = f(K_i)$ where U_i is the utility function for asset owner type *i*.

Asset owner *i*'s problem is to maximize his utility by choosing C_{i0}^{farmer} and A_i . Each type of asset owner's optimization problem and budget constraints are shown in equation (3.1.1) and (3.1.2) respectively.

$$Max_{\{C_{i0}^{farmer}, A_i\}} \quad U\left(C_{i0}^{farmer}\right) + EV\left[W_{i1}\right] \tag{3.1.1}$$

Where

$$W_{i1} = \underbrace{\left\{W_{i0} - C_{i0}^{farmer} - A_i\left(P_{opt} + S\right)\right\}\left(1 + r_f\right)}_{\text{Left-Over Initial Endowment}} + \underbrace{A_i \max\left[k_i - \tilde{P}_1, 0\right]}_{\text{Options payoff}} + \underbrace{Q_i \tilde{P}_1}_{\text{Additional Endowment}} \tag{3.1.2}$$

From equation (3.1.1), $U(.)$ is utility at period 0 and $EV[.]$ is expected utility at period 1. The consumption at period 1 is comprised of: 1) The left-over initial endowment, 2) the options payoff, and 3) the additional endowment at period 0. The left-over initial endowment is the left-over endowment after the asset owner allocates his initial endowment (W_{i0}) into consumption (C_{io}^{farmer}) and purchases A_i options at the price $\left(P_{opt} + S\right)$ where P_{opt} is the theoretical option price derived from the BSM, and S is the additional price of the option (the bias). This left-over endowment is invested to receive a rate of return $r_f\%$ per year[22].

Dropping the asset owner's superscript and solving the optimization problem using the Lagrangian method, First Order Conditions (FOCs) become

C_0^{farmer} :

$$U'\left(C_0^{farmer}\right)+E\left[V'(W_1)\frac{dW_1}{dC_0^{farmer}}\right]=U'\left(C_0^{farmer}\right)-E\left[V'(W_1)(1+r_f)\right]=0 \qquad (3.1.3)$$

i.e.

$$U'\left(C_0^{farmer}\right)=E\left[V'(W_1)(1+r_f)\right] \qquad (3.1.4)$$

A:

$$E\left[V'(W_1)\frac{dW_1}{dA}\right]=0 \qquad (3.1.5)$$

Equation (3.1.5) can be written as

$$E\left[V'(W_1)\left(\max\left[k-\tilde{P}_1,0\right]-\left(P_{opt}+S\right)\left(1+r_f\right)\right)\right]=0 \qquad (3.1.6)$$

$$E\left[V'(W_1)\left(\max\left[k-\tilde{P}_1,0\right]\right)\right]=\left(P_{opt}+S\right)*E\left[V'(W_1)\left(1+r_f\right)\right] \qquad (3.1.7)$$

Plug in FOC from C_0

$$E\left[V'(W_1)\left(\max\left[k-\tilde{P}_1,0\right]\right)\right]=\left(P_{opt}+S\right)*U'\left(C_0^{farmer}\right) \qquad (3.1.8)$$

Note that with the exponential utility used in Brennan, utility function takes the form of

$$U(W_1)=\left(-\frac{1}{a}\right)\exp(-a*W_1) \qquad (3.1.9)$$

[22] The rate of return can be negative in case of carrying cost of future goods and can be positive in case of opportunity cost of capital.

$$U_x'(W_1) = \exp(-a*W_1)\frac{dW_1}{dx} \qquad (3.1.10)$$

Therefore, equation (3.1.3) becomes

$$\begin{aligned}&\exp\left(-aC_0^*\right)\\ &-E\left[\exp\left((-a)\left\{\left\langle W_0 - C_0^* - A^*\left(P_{opt} + S^*\right)\right\rangle\left(1+r_f\right) + A^*\max\left[k-\tilde{P}_1,0\right] + Q\tilde{P}_1\right\}\right)*\left(1+r_f\right)\right] = 0\end{aligned} \qquad (3.1.11)$$

$$\begin{aligned}&\frac{\exp\left(-aC_0^*\right)}{\left(1+r_f\right)}\\ &= E\left[\exp\left((-a)\left\{\left\langle W_0 - C_0^* - A\left(P_{opt} + S^*\right)\right\rangle\left(1+r_f\right) + A^*\max\left[k-\tilde{P}_1,0\right] + Q\tilde{P}_1\right\}\right)\right]\end{aligned} \qquad (3.1.12)$$

and equation (3.1.5) becomes

$$E\begin{bmatrix}\exp\left((-a)\left\{\left\langle W_0 - C_0^* - A^*\left(P_{opt} + S^*\right)\right\rangle\left(1+r_f\right) + A^*\max\left[k-\tilde{P}_1,0\right] + Q\tilde{P}_1\right\}\right)\\ *\left(-\left(P_{opt} + S^*\right)\left(1+r_f\right) + \max\left[k-\tilde{P}_1,0\right]\right)\end{bmatrix} = 0 \qquad (3.1.13)$$

separating the terms that are known with certainty,

$$\begin{aligned}&\exp\left((-a)\left\{\left\langle W_0 - C_0^* - A^*\left(P_{opt} + S^*\right)\right\rangle*\left(1+r_f\right)\right\}\right)\\ &*E\left[\exp\left((-a)\left\{A^*\max\left[k-\tilde{P}_1,0\right] + Q\tilde{P}_1\right\}\right)*\left(\max\left[k-\tilde{P}_1,0\right] - \left(P_{opt} + S^*\right)*\left(1+r_f\right)\right)\right] = 0\end{aligned} \qquad (3.1.14)$$

cancelling out the exponential term that is known with certainty and rearranging some terms,

$$E\left[\exp\left((-a)\left\{A^*\max\left[k-\tilde{P}_1,0\right] + Q\tilde{P}_1\right\}\right)*\left(\max\left[k-\tilde{P}_1,0\right] - \left(P_{opt} + S^*\right)*\left(1+r_f\right)\right)\right] = 0 \qquad (3.1.15)$$

$$E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)*\max\left[k-\tilde{P}_1,0\right]\right]$$
$$=E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)*\left(\left(P_{opt}+S^*\right)*\left(1+r_f\right)\right)\right] \tag{3.1.16}$$

$$E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)*\max\left[k-\tilde{P}_1,0\right]\right]$$
$$=\left(P_{opt}+S^*\right)*\left(1+r_f\right)*E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)\right] \tag{3.1.17}$$

The last equality results because the option price is taken as exogenous from the BSM and, hence, is known with certainty at period 0. The bias demand function ($S^{farmer*}$) becomes

$$S^{farmer*}=\frac{E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)*\max\left[k-\tilde{P}_1,0\right]\right]}{E\left[\exp\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)\right]}*\frac{1}{\left(1+r_f\right)}-P_{opt} \tag{3.1.18}$$

By the lognormality assumption, we have $\tilde{P}_1 \sim \log Normal\left(\ln(P_0)+\left(r_f-\frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)$.

Therefore, the probability function of $\tilde{P}_1$ can be written as

$$f(P_1)=\frac{1}{\sqrt{2\pi}\sigma\sqrt{T}P_1}\exp\left(\left(-\frac{1}{2}\right)\frac{\left[\ln(P_1)-\ln(P_0)-\left(r_f-\frac{\sigma^2}{2}\right)T\right]^2}{\left(\sigma\sqrt{T}\right)^2}\right)$$

The asset owner's demand function is then

$$S^{farmer*}=\frac{M_1}{M_2}*\frac{1}{\left(1+r_f\right)}-P_{opt} \tag{3.1.19}$$

Where

$$M_1=\frac{1}{\sqrt{2\pi}\sigma\sqrt{T}P_1}$$
$$*\int_0^\infty \max\left[k-\tilde{P}_1,0\right]\exp\left(\left((-a)\left\{A^* \max\left[k-\tilde{P}_1,0\right]+Q\tilde{P}_1\right\}\right)-\frac{1}{2}\frac{\left[\ln(P_1)-\ln(P_0)-\left(r_f-\frac{\sigma^2}{2}\right)T\right]^2}{\left(\sigma\sqrt{T}\right)^2}\right)dP_1 \tag{3.1.20}$$

$$M_2 = \frac{1}{\sqrt{2\pi}\sigma\sqrt{T}P_1}$$

$$* \int_0^\infty \exp\left(\left((-a)\left\{ A^* \max\left[k - \tilde{P}_1, 0 \right] + Q\tilde{P}_1 \right\} \right) - \frac{1}{2} \frac{\left[\ln(P_1) - \ln(P_0) - \left(r_f - \frac{\sigma^2}{2} \right) T \right]^2}{\left(\sigma\sqrt{T} \right)^2} \right) dP_1 \tag{3.1.21}$$

3.2 The Option writer's problem

The option writer operates in the monopoly market. For simplicity, the option writer is assumed to be a risk-neutral agent. For such a risk-neutral agent, his objective of maximizing his total expected utility is equivalent to maximizing his total expected profit. Note that if the option writer is assumed to have the usual concave utility function, the analysis will remain the same but his decision regarding the amount of the options and the price of the bias will be different. The option writer's total revenue derives from the sales of options. In return, he also incurs fixed cost and variable cost resulting from selling options. His fixed cost can be thought as the start up costs such as the payment for the right to use the seat on the exchange and his variable cost can be thought as the cost of hedging his portfolio. In period 1, once the price of the asset is known, the option writer is also responsible for the payoff of the options he sold. Following the standard monopoly market framework, the monopolist views the market demand as his own demand. His maximization problem can be formulated as

$$E[\Pi_{writer}] = \sum_{i=1}^{n} \left(P_{opt} + S(A_i) \right) A_i - H - \sum_{i=1}^{n} cA_i - E\left[\sum_{i=1}^{n} \min\left(\tilde{P}_1 - k_i, 0 \right) A_i \right] \left(\frac{1}{1+r_f} \right) \tag{3.2.1}$$

where $S(A_i)$ follows from (3.1.19) and $i = 1,2,3,\ldots n^{th}$ asset owner. c represents the marginal cost and H represents the fixed cost. By substituting the demand from the asset owner, the above maximization problem can be written as

$$E[\Pi_{writer}] = \sum_{i=1}^{n} \left(P_{opt} + \frac{M_1(A)}{M_2(A)} * \frac{1}{(1+r_f)} - P_{opt} \right) A_i - H - \sum_{i=1}^{n} cA_i$$
$$- E\left[\sum_{i=1}^{n} \min\left(P_1 - k_i, 0 \right) A_i \right] \left(\frac{1}{1+r_f} \right) \tag{3.2.2}$$

The solution for A_i is characterized by the following FOC:

$$A_i:\ \frac{d\left(\left(P_{opt}+\frac{M_1(A)}{M_2(A)}*\frac{1}{(1+r_f)}-P_{opt}\right)A_i\right)}{dA_i}=c+E[\min(P_1-k_i,0)]\left(\frac{1}{1+r_f}\right)\quad,\quad for\ i=1,2,\ldots n \tag{3.2.3}$$

This equation can be thought of as representing a situation in which the monopolist equates his marginal revenue to his marginal cost.

3.3 Equilibrium

To analyze market equilibrium, I consider two different cases. The first case assumes that the option writer operates in a market where he is unable to impose different biases on different asset owners, denoted as a "no discrimination" case. This is similar to the classic monopoly market where the monopolist cannot conduct any discrimination among his patrons, i.e., options with different strike prices are charged the same bias (S). On the other hand, the second case, denoted as a "perfect discrimination" case, allows the option writer to charge different biases for each asset owner. In this case, he treats each individual asset owner's demand as one of different demands from different markets and maximizes the profit for each asset owner separately.

Case I: No Discrimination Case

In the no-discrimination case, the option writer is unable to practice price discrimination among asset owners, i.e. he has to charge the same level of bias to all asset owners. This case can be thought of as one in which the option dealer is charging a flat fee for each option regardless of strikes and quantity or the case where he is regulated by the government and is only allowed to charge a certain amount of fee across asset owners. The option writer's total revenue is the summation of BSM price and the bias that he charges each asset owner. The total cost of the option writer is composed of fixed cost and variable cost. The fixed cost represents costs that he has incurred, such as the exchange seat and opportunity cost, in order to become an option writer. The variable cost represents costs associated with each individual option transaction, e.g., dynamic hedging costs that increase with the quantity of options sold. In equilibrium, the optimal quantity sold to each asset owner is characterized by equation (3.2.3) with the additional condition that the additional charge (bias) is equal among asset owners, that is

$$S_i^{farmer*} = S_j^{farmer*} \quad where \quad i, j = 1,2,.....n \tag{3.3.1}$$

with $S_i^{farmer*}$ defined by equation (3.1.19), (3.1.20) and (3.1.21). Although there is no closed-form solution for equation (3.3.1), the model suggests that the optimal bias (S^*) can be written as a function of the following parameters

$$S^* = f\left(a, k, P_0, \sigma^2, r_f, P_{opt}\left(r_f, k, P_0, \sigma^2, T\right), Q, c\right) \tag{3.3.2}$$

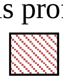
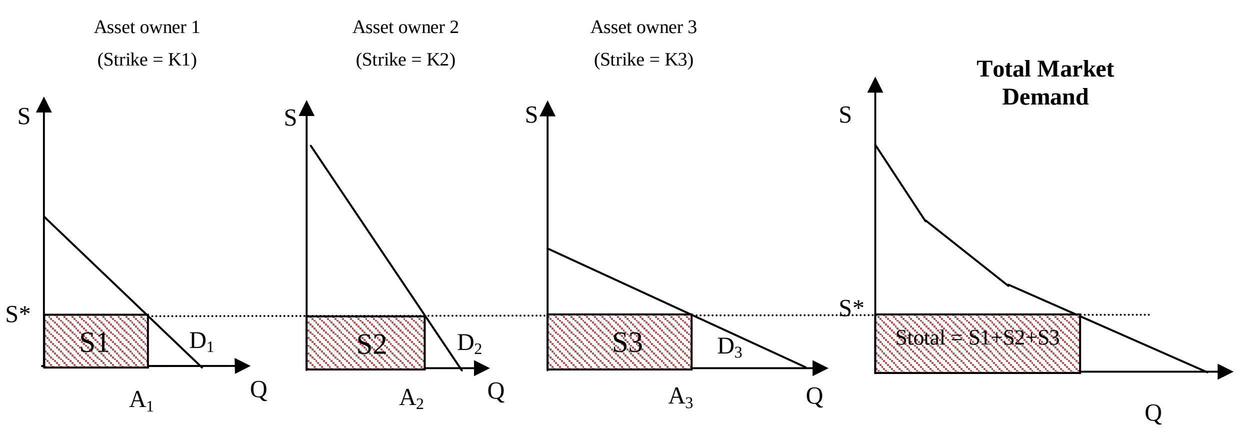
The following diagram graphically illustrates the option writer's maximization problem. The diagram assumes zero fixed cost, zero variable cost and three asset owners in the market. Each asset owner exhibits linear demand. The right-hand-side plot represents the total market demand. To maximize his profit, the option writer chooses the optimal level of bias (S^*) represented by the shaded area  . By varying the bias level, the option writer can find the bias level that provides him or her with the greatest profit level, i.e., the largest shaded area. The result produced by this numerical method is presented in the next section.

Figure 1. Demands By Strikes: Case I No Discrimination



Case II: The Perfect Discrimination Case

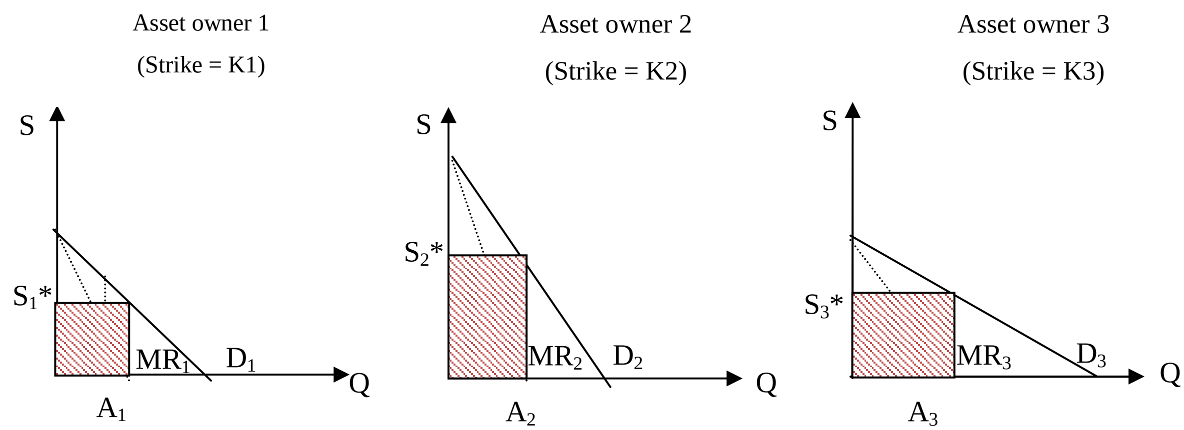
In contrast to the no-discrimination case, the option writer in the perfect-discrimination case can charge different level of biases to different asset owners. In this case, the option writer treats each asset owner as if he or she followed an individual demand curve. The option writer is maximizing his utility by maximizing the profit from each demand curve. With the same cost structure as the no-discrimination case, the option writer's maximization problem is shown in (3.2.1)and can be broken down into maximizing profit for each asset owner's demand, i.e.,

$$\Pi_{writer} = \sum_{i=1}^{n} \Pi_i \tag{3.3.3}$$

$$\begin{aligned}\Pi_{writer} = &\left[\left(P_{opt} + S_1(A_1)\right)A_1 - cA_1 - E\left[\min\left(\tilde{P}_1 - k_1, 0\right)A_i\right]\right] \\ &+ \left[\left(P_{opt} + S_2(A_2)\right)A_2 - cA_2 - E\left[\min\left(\tilde{P}_1 - k_2, 0\right)A_i\right]\right] \\ &+ \text{K} + \left[\left(P_{opt} + S_n(A_n)\right)A_n - cA_n - E\left[\min\left(\tilde{P}_1 - k_3, 0\right)A_i\right]\right] \\ &- H\end{aligned}$$

Similar to case (1), the solution is characterized by equation (3.2.3). However, since the monopolist has the ability to charge different bias for different asset owners, he is not subject to the equality constraint given by equation (3.3.1). A closed form solution for S_i^* does not exist. However, S_i^* will be a function of the set of variables shown in equation (3.3.2). Graphically, the model can be represented b y the following diagram:

Figure 2. Demands By Strikes: Case II Perfect Discrimination



The shaded area represents the profit that the option writer is able to make from each types of asset owner. As mentioned earlier, the only difference between each type of the asset owners is the preference on the strike prices. By choosing the optimal bias level corresponding to the demand of each asset owner type, the monopolist conducts his or her pricing strategy by maximizing each shaded area separately.

In the models presented above, the option writer imposes an additional cost or "bias" on options sold to asset owners. As described in the literature, this bias accounts for imperfections in the BSM, such as the violation of several model assumptions such as constant volatility and frictionless markets. However, when pricing options using the BSM or its variations [20], instead of adjusting in terms of dollar amounts, practitioners typically adjust the volatility, the only unknown input in the model, to take into account factors that the model fails to represent. The implied volatility inverted from the options prices is thus contaminated by this adjustment. The next section presents a numerical analysis of this theoretical model.

3.4 Numerical Analysis

This section performs a numerical analysis of the theoretical model using Matlab. First, I plot the asset owners' option demands. Next, I simulate model equilibrium for both of the cases discussed above: (i) the no-discrimination case and (ii) the perfect-discrimination case. The code used in this section is provided in Appendix A.

Demand for Options

Option demand ($S_i^{farmer*}$) is defined by equations (3.1.19), (3.1.20) and (3.1.21). Using the following initial values: a = 1, P_0 (or F_0 for futures) = 1, σ (sigma) = 0.25, T = 1, r_f = 0.05, Q = 10, Figure 3 shows the demands derived from different types of asset owner with various strike preferences. Since the demand function involves integration over the density function of the future price, a numerical integration technique is employed. This is accomplished by dividing the range of possible values into small trapezoids. Integration over a certain range of values is performed by the area summation of these small trapezoids. A preference for a higher strike results in a steeper demand curve than that for a lower strike. This means that, since an asset owner generally prefers higher strikes for put options, the amount of bias that he or she is willing to pay increases.

Figure 3. Farmer Demands (Base Case)

Base case (a = 1, F_0 = 1, μ=1, σ = 0.25, T = 1, r_f = 0.05, Q = 10)

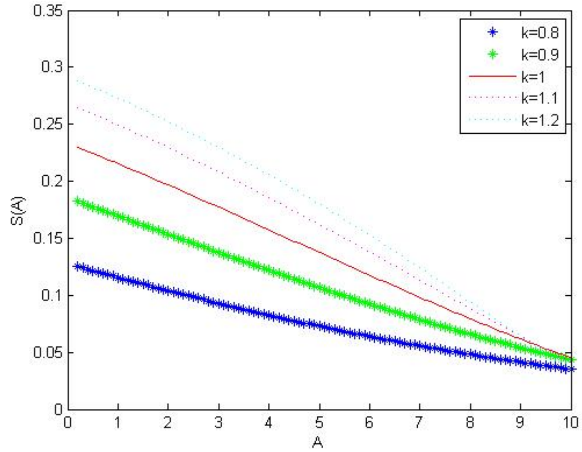
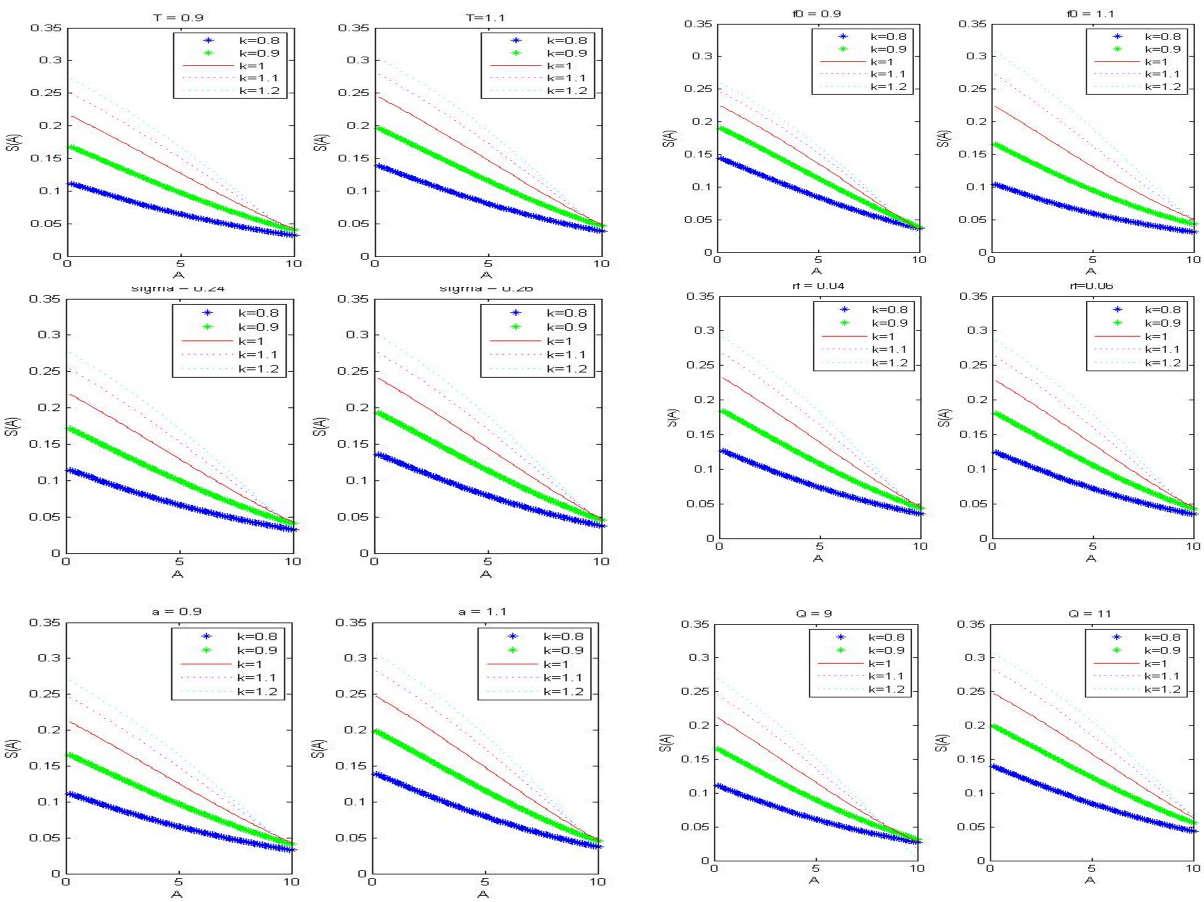


Figure 4 shows sensitivity of the demand curve. Each plot shows the change of demands with respect to the change in each parameter while holding other parameters constant. The decrease (increase) in *a*, *T*, σ, Q and F_0 results in a demand curve tilting downward (upward). To interpret this result, we can use the change in the risk-aversion coefficient (*a*) as an example. Such a decrease in the this coefficient means that the asset owner has become less risk-averse and therefore is less willing to pay for the surcharge or bias from purchasing the options. Therefore, at the same value for which he is willing to purchase the options, his willingness to pay for the bias decreases; hence, his demand tilts downward from its original position. The impact of the risk-free rate appears to be the opposite, i.e., a decrease (increase) in the risk-free rate results in the demand curve tilting slightly upward (downward). This can be explained by considering the opportunity cost of money. When the opportunity cost of money decreases, the asset owner becomes more willing to pay for the surcharge, as indicated by the asset owner's demand tilting upward.

Figure 4. Asset Owner Demands Using Different Parameters



Equilibrium Analysis

To analyze the model equilibrium, I assume initial parameters similar to those of the previous section. I further assume that the market consists of three asset owners with respective strike preferences of 0.9, 1.0, and 1.1. With current future prices (F_0) at 1, these strikes can be thought of as the put options with 90%, 100% and 110% strikes, i.e., the options are out-the-money, at-the-money and in-the-money respectively. The monopolist's profit maximization problem follows Equation (3.2.1), where $S_i^{farmer*}$ is defined by equation (3.1.19), (3.1.20) and (3.1.21). The monopolist simultaneously chooses A_i^* for i = 1, 2, 3 to maximize his expected profit function in (3.2.1). However, since each asset owner is facing a physical constraint, he is not allowed to purchase more put options than the quantity he is endowed with(Q). Therefore, each asset owner is facing additional constraints as follows

$$A_i^* \leq Q_i \qquad , \quad for \quad i = 1,2,3 \tag{3.4.1}$$

The optimization problem searches for the optimal option quantity sold to each asset owner ($A_i^* \quad for\ i = 1,2,3$) that will maximize the monopolist's expected profit function, the objective function. The optimal level of bias is then derived by substituting A_i^* into the S_i^* equation in (3.1.19), (3.1.20) and (3.1.21).

To complete this task, I employ the non-linear constraint function *fmincon* search algorithm in Matlab. This function searches for the value of A_i^* that minimizes the objective function. Therefore, to apply this function to the problem, I multiply the monopolist's objective function by -1. Hence, the objective function becomes a global minimization problem.

The equilibrium analysis is done for two separate cases: 1) the no-discrimination case, and 2) the perfect-discrimination case. The no-discrimination case requires additional constraints that restrict the optimal bias (S_i^*) to be equal for all asset owners, i.e.,

$$S_i^* = S_i^* \qquad , \quad for \quad i \neq \mathrm{j} \tag{3.4.2}$$

Table 1. Equilibrium Analysis: Case I No Discrimination

Parameters	Base Value	H change		c change		a change		f_0 change		σ change		r_f change		Q change		T change	
H	1	0.9	1.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c	0.1	0.1	0.1	0.09	0.11	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
a	1	1	1	1	1	0.9	1.1	1	1	1	1	1	1	1	1	1	1
f_0	1	1	1	1	1	1	1	0.9	1.1	1	1	1	1	1	1	1	1
σ	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.24	0.26	0.25	0.25	0.25	0.25	0.25	0.25
r_f	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.06	0.05	0.05	0.05	0.05
Q	10	10	10	10	10	10	10	10	10	10	10	10	10	9	11	10	10
T	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9	1.1
K = 0.8 , 0.9 , 1.0 , 1.1 , 1.2																	
Result																	
S	**0.063**	0.063	0.063	0.057	0.069	0.084	0.072	0.039	0.095	0.084	0.065	0.063	0.062	0.054	0.072	0.081	0.066
A (K = 0.8)	**6.137**	6.137	6.137	6.883	5.463	2.880	5.755	9.711	1.031	3.139	6.468	6.152	6.122	5.945	6.330	3.107	6.504
A (K = 0.9)	**8.259**	8.259	8.259	8.781	7.777	6.027	7.872	10.000	5.030	6.214	8.379	8.268	8.249	7.843	8.659	6.228	8.393
A (K = 1.0)	**8.936**	8.936	8.936	9.297	8.597	7.375	8.633	9.806	7.029	7.510	8.995	8.941	8.930	8.354	9.496	7.522	9.012
A (K = 1.1)	**9.069**	9.069	9.069	9.331	8.820	7.906	8.846	9.580	7.922	8.004	9.118	9.072	9.067	8.389	9.730	8.002	9.144
A (K = 1.2)	**9.044**	9.044	9.044	9.248	8.848	8.109	8.876	9.416	8.289	8.186	9.094	9.045	9.043	8.308	9.764	8.168	9.128
Total A	**41.444**	**41.444**	**41.444**	**43.540**	**39.504**	**32.297**	**39.981**	**48.512**	**29.302**	**33.052**	**42.054**	**41.478**	**41.411**	**38.839**	**43.979**	**33.027**	**42.179**
Profit	**7.362**	**7.362**	**7.362**	**7.787**	**6.957**	**6.917**	**7.530**	**11.625**	**4.475**	**6.778**	**7.799**	**7.433**	**7.293**	**6.383**	**8.383**	**6.660**	**7.902**
Change S		0.000	0.000	-0.006	0.006	0.021	0.009	-0.024	0.032	0.021	0.002	0.001	0.000	-0.009	0.009	0.019	0.003
Change Total A		0.000	0.000	2.096	-1.940	-9.147	-1.463	7.068	-12.142	-8.392	0.610	0.033	-0.033	-2.605	2.535	-8.418	0.735
Change Total F		0.000	0.000	0.425	-0.405	-0.445	0.168	4.263	-2.888	-0.584	0.437	0.071	-0.069	-0.979	1.021	-0.702	0.540

Table 2. Equilibrium Analysis: Case II Perfect Discrimination

Parameters	Base Value	H change		c change		a change		f_0 change		σ change		rf change		Q change		T change	
H	1	0.9	1.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c	0.1	0.1	0.1	0.09	0.11	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
a	1	1	1	1	1	0.9	1.1	1	1	1	1	1	1	1	1	1	1
f_0	1	1	1	1	1	1	1	0.9	1.1	1	1	1	1	1	1	1	1
σ	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.24	0.26	0.25	0.25	0.25	0.25	0.25	0.25
r_f	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.06	0.05	0.05	0.05	0.05
Q	10	10	10	10	10	10	10	10	10	10	10	10	10	9	11	10	10
T	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9	1.1
K = 0.8 , 0.9 , 1.0 , 1.1 , 1.2																	
Result																	
S (K = 0.8)	**0.091**	0.091	0.091	0.085	0.096	0.084	0.097	0.077	0.092	0.088	0.093	0.091	0.090	0.084	0.097	0.087	0.094
S (K = 0.9)	**0.086**	0.086	0.086	0.079	0.092	0.077	0.095	0.051	0.102	0.084	0.087	0.086	0.085	0.077	0.094	0.083	0.088
S (K = 1.0)	**0.061**	0.061	0.061	0.053	0.068	0.048	0.073	0.035	0.096	0.058	0.063	0.061	0.060	0.048	0.073	0.057	0.064
S (K = 1.1)	**0.041**	0.041	0.041	0.041	0.041	0.040	0.043	0.028	0.073	0.039	0.044	0.042	0.041	0.040	0.043	0.038	0.045
S (K = 1.2)	**0.034**	0.034	0.034	0.034	0.034	0.033	0.036	0.021	0.048	0.032	0.037	0.035	0.034	0.033	0.036	0.030	0.039
A (K = 0.8)	**3.241**	3.241	3.241	3.791	2.728	2.955	3.459	5.698	1.328	2.708	3.717	3.280	3.203	2.660	3.804	2.545	3.815
A (K = 0.9)	**6.488**	6.488	6.488	6.967	6.042	6.653	6.353	9.113	4.465	6.199	6.745	6.510	6.467	5.987	6.990	6.110	6.797
A (K = 1.0)	**9.067**	9.067	9.067	9.528	8.651	9.668	8.597	10.000	6.952	9.005	9.119	9.067	9.067	8.701	9.456	8.985	9.130
A (K = 1.1)	**10.000**	10.000	10.000	10.000	10.000	10.000	10.000	10.000	8.941	10.000	10.000	10.000	10.000	9.000	11.000	10.000	10.000
A (K = 1.2)	**10.000**	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	9.000	11.000	10.000	10.000
Total A	**38.797**	**38.797**	**38.797**	**40.285**	**37.422**	**39.275**	**38.409**	**44.812**	**31.687**	**37.912**	**39.581**	**38.857**	**38.737**	**35.348**	**42.250**	**37.641**	**39.741**
Profit	**7.631**	**7.631**	**7.631**	**8.026**	**7.250**	**7.493**	**7.769**	**11.959**	**4.588**	**7.225**	**8.045**	**7.700**	**7.563**	**6.644**	**8.646**	**7.112**	**8.138**
Change Total A		0.000	0.000	1.488	-1.375	0.479	-0.388	6.015	-7.110	-0.885	0.784	0.060	-0.059	-3.449	3.454	-1.156	0.944
Change Total F		0.000	0.000	0.395	-0.381	-0.138	0.138	4.329	-3.043	-0.406	0.414	0.069	-0.068	-0.987	1.015	-0.519	0.507

Table 1and Table 2 show the numerical analysis results for the no-discrimination case and the perfect -discrimination case, respectively. In the perfect discrimination case, a monopolist maximizes his expected profit by charging an asset owner with a higher strike preference (in-the-money (ITM) put option) a lower bias price in order to boost their hedged quantities. The hedged quantity is higher among these asset owners than that for the lower strike (out-the-money (OTM) put option) asset owners. In all scenarios, the monopolist's ability to set different levels of bias for different asset owners allows him to increase his profit. In the base case, this ability to discriminate increases his profit from \$7.36 to \$7.63.

The result of the sensitivity analysis is similar in both cases. As we should expect for a risk-neutral decision maker, the change of fixed cost (H) should be viewed as a sunk cost that turns out to have no impact on the quantity hedged and the level of bias. A decrease in variable cost (c) leads to a decrease in the level of bias and therefore an increase in the quantity hedged. Similarly, a decrease in future price (F_0) and risk-free rate (r_f) leads to a smaller bias level and a higher quantity hedged. However, decreases in volatility (σ), endowment (*Q*), and time (*T*) produce an opposite result. Finally, a decrease in the risk-aversion coefficient (*a)* results in a higher quantity hedged and lower profit in the perfect-discrimination case and a lower quantity hedged and lower profit in the no-discrimination case.

The bias resulting from the model can produce a volatility smile. To show this, I calculate the prices of put options using similar parameters as those in the base case, i.e., F_0 = 1, r_f = 5%, T = 1, σ = 25% and k = 0.8, 0.9, 1.0, 1.1, and 1.2. The second column in Table 3 and Table 4 shows the BSM prices for the no-discrimination case and the perfect-substitution case, respectively. In the top panel, the third column shows the bias level calculated using parameters from the base case as shown in Table 1 and Table 2. On the middle and last panels, the third column shows the bias level calculated from the case where the variable cost (c) = 0.9 and the risk aversion coefficient (a) = 1.1, respectively. The market price paid by asset owners is the summation of the BSM price and the bias level shown in column four. Finally, the implied volatility in column four is calculated by searching for the volatility that equates the BSM price to this market price.

Figure 5. Implied Volatility Across Moneyness: Case I No Discrimination

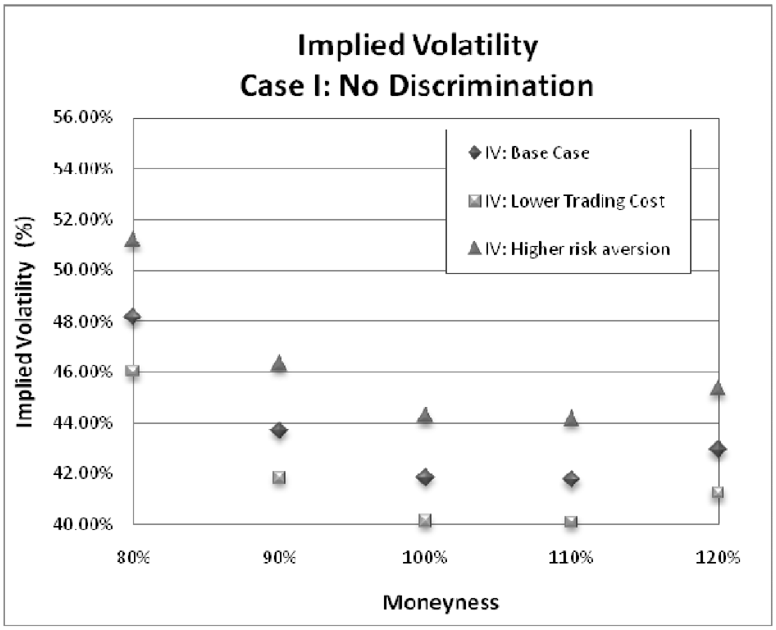


Figure 5 shows that when a constant bias adjustment is applied to the BSM price, the implied volatility smile can be generated. A decrease in trading-variable cost leads to a decrease in the bias and a downward shift of the implied volatility curve. As asset owners become more risk averse, i.e., as the risk aversion coefficient increases from 1 to 1.1, the bias increases, leading to an upward shift in the implied volatility curve.

The volatility curve behaves differently for the perfect discrimination case. As shown in Figure 6, the volatility curve exhibits a higher bias for an 80% strike and a lower bias for a 120% strike. Moreover, the volatility curve slopes downward. As the cost of trading decreases, the volatility curve tilts downward with a larger decrease in volatility for an 80% strike. The decrease in variable cost doesn't impact the higher strikes (110% and 120%) that already have a lower bias amount. This could also be driven by the profit-maximizing problem of the monopolist in which he tries to maintain a lower amount of bias for these asset owners in order to increase their quantity hedged. As an asset owner becomes more risk-averse, he is willing to pay the higher bias represented by the upward tilt of the implied volatility curve.

Figure 6. Implied Volatility Across Moneyness: Case II Perfect Discrimination

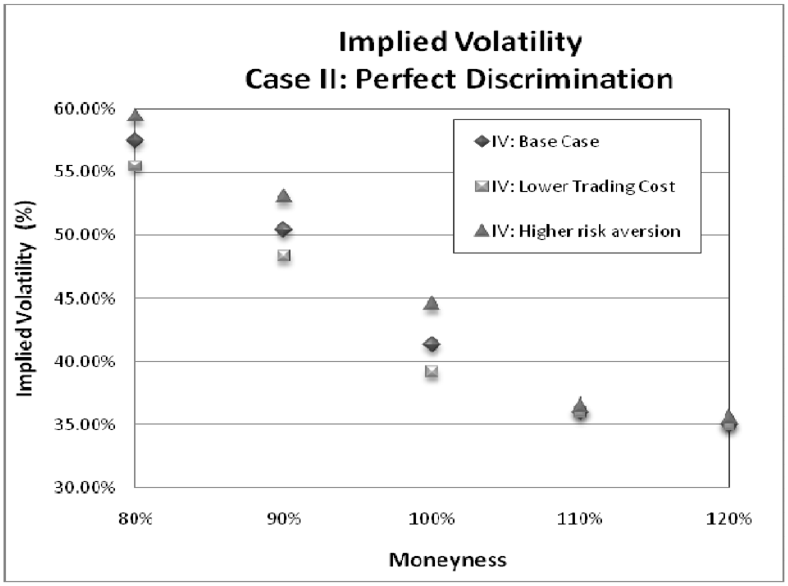


Table 3. Implied Volatility Calculation: Case I No Discrimination

K	BSM Price	Bias (S)	Base Market Price	**IV: Base Case**
80%	0.022	0.063	0.085	**48.16%**
90%	0.050	0.063	0.113	**43.69%**
100%	0.095	0.063	0.158	**41.84%**
110%	0.154	0.063	0.217	**41.77%**
120%	0.225	0.063	0.288	**42.94%**

K	BSM Price	Bias (S)	Trading Cost = 0.09 Market Price	**IV: Lower Trading Cost**
80%	0.022	0.057	0.078	**46.03%**
90%	0.050	0.057	0.107	**41.84%**
100%	0.095	0.057	0.151	**40.14%**
110%	0.154	0.057	0.211	**40.11%**
120%	0.225	0.057	0.282	**41.24%**

K	BSM Price	Bias (S)	Risk Aversion Coefficient = 1.1 Market Price	**IV: Higher risk aversion**
80%	0.022	0.072	0.094	**51.19%**
90%	0.050	0.072	0.122	**46.33%**
100%	0.095	0.072	0.167	**44.27%**
110%	0.154	0.072	0.226	**44.14%**
120%	0.225	0.072	0.297	**45.35%**

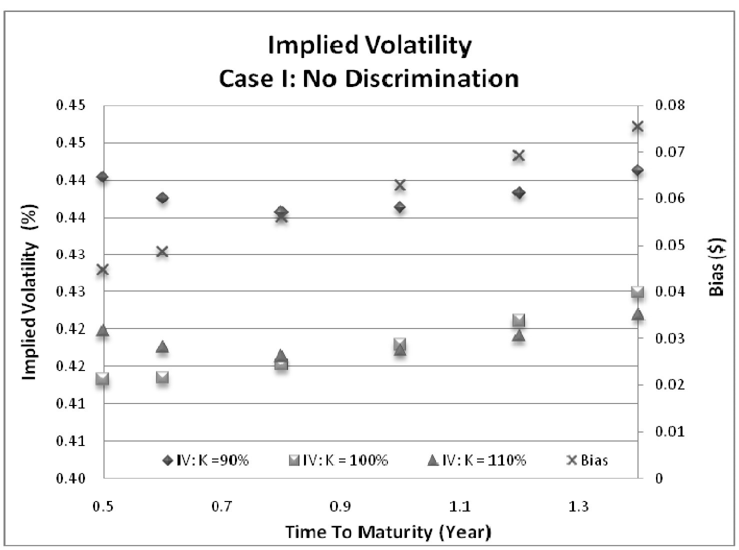
Table 4. Implied Volatility Calculation: Case II Perfect Discrimination

K	BSM Price	Bias	Base Market Price	**IV: Base Case**
80%	0.022	0.091	0.113	**57.51%**
90%	0.050	0.086	0.136	**50.43%**
100%	0.095	0.061	0.156	**41.30%**
110%	0.154	0.041	0.195	**35.96%**
120%	0.225	0.034	0.259	**35.00%**

K	BSM Price	Bias	Trading Cost = 0.09 Market Price	**IV: Lower Trading Cost**
80%	0.022	0.085	0.107	**55.52%**
90%	0.050	0.079	0.129	**48.38%**
100%	0.095	0.053	0.148	**39.15%**
110%	0.154	0.041	0.195	**35.96%**
120%	0.225	0.034	0.259	**35.00%**

K	BSM Price	Bias	Risk Aversion Coefficient = 1.1 Market Price	**IV: Higher risk aversion**
80%	0.022	0.097	0.119	**59.49%**
90%	0.050	0.095	0.145	**53.07%**
100%	0.095	0.073	0.168	**44.54%**
110%	0.154	0.043	0.197	**36.49%**
120%	0.225	0.036	0.261	**35.56%**

Figure 7. Implied Volatility Across Time-to-Maturity: Case I No Discrimination



Furthermore, the partial equilibrium model also allows the implied volatility to be different for different times to maturity. Figure 7 and Figure 9 show the plots of implied volatilities across time to maturity ranging from 0.5 years to maturity (approximately 120 business days) to 1.3 years to maturity (approximately 312 business days). For the no discrimination case, the option writer is forced to charge the same amount of bias to all asset owners. The plot of the bias is shown in Figure 7 using the right Y-Axis. Although the bias is increasing with time to maturity, the implied volatility exhibits a smile across the time to maturity range. As shown in Figure 7, the implied volatility for the options with 90% and 110% moneyness decreases when time to maturity is less than 0.8 year and increases afterward. The implied volatility of the ATM option exhibits increasing implied volatility as time to maturity increases from 0.5 years to 1.3 years

In the perfect-discrimination case, the option writer has the ability to charges different biases to different asset owners.

Figure **8** shows different levels of bias across time. As we might expect, for put options, at a fixed time to maturity, the bias is lowest for ITM put options and highest for OTM put options. As time to maturity increases, the bias increases at a decreasing rate for all moneyness levels. The implied volatilities inverted from the summation of the bias and the BSM are plotted in Figure 9. The implied volatility of options with 90% moneyness decreases as time to maturity increases. A mild smile curve is observed for ATM options and an increase in volatility is observed for a 110% moneyness option.

In order to explain the non-constant implied volatility shapes (both across the strikes and times to maturity), two points should be considered. First, examine the behavior of the option vega[23], one of the most important option greeks. Vega is particularly important when calculating the implied volatility. Figure 10 shows the vega profile across moneyness and time to maturity. An option displays higher vega as time to maturity increases. At the same time to maturity, an ATM strike (or, slightly, an ITM strike as time to maturity increases) displays the highest vega value.

Second, different amounts of bias (e.g. bias shown in Table 3, Table 4, Figure 7 and Figure 8). are derived based on demand and supply in the market. Hence, depending on the moneyness level and the time to maturity, the implied volatility can exhibit different shapes.

To explain how these two forces work together, assume that a bias amount of $1 is added to the option price to get the total option cost. The implied volatility of the total option cost is the

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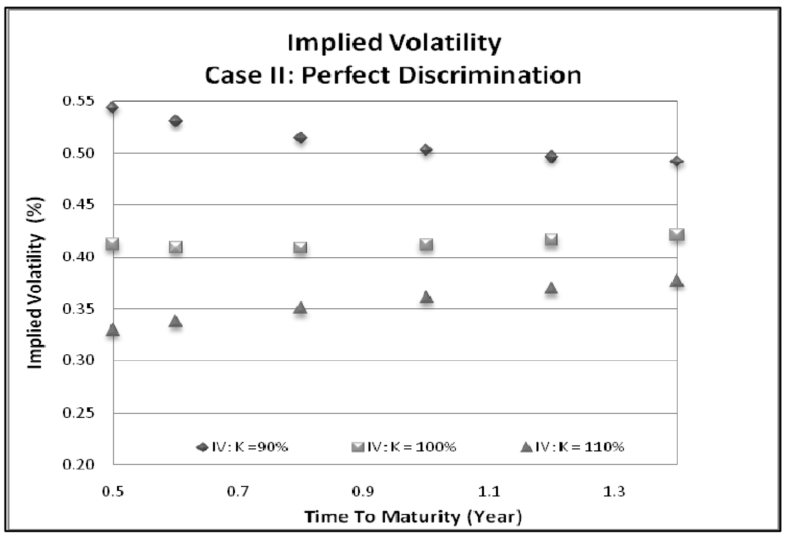
[23] Vega of an option measures the change in the price of the option with respect to the change in the volatility of the underlying asset. For this study's purpose, vega is defined per 100 basis points of volatility, i.e., vega = 0.0045 means that a change 1% of volatility will result in a 0.0045$ change of the option value.

volatility that equates the BSM option price to the total option cost. As shown in Table 3, options with higher vega have lower implied volatility, in contrast to options with lower vega. Options can have higher vega depending on their moneyness and times to maturity. An ATM will have higher vega than a deep OTM option, or longer time to maturity options will have higher vega than options that are maturing soon. Therefore, the shape of the implied volatility will depend on the option price, the option moneyness level, the time to maturity, and the bias imposed on the option.

Figure 8. Bias Across Time-to-Maturity: Case II Perfect Discrimination



To show how the bias amount and the option vega work together, let's consider the shape of the options with 90% strike across a range of different times to maturity. As shown in Figure 10, for the same strike, as time to maturity increases, so does the option vega. First, consider the perfect discrimination case. As option vega increases with time to maturity, the bias amount also increases, although not fast enough to offset the increase in vega. As a result, the implied volatility appears to decrease with time to maturity. However, for the no discrimination case, a similar situation occurs for options when time to maturity is less than 0.8 year. However, once the time to maturity exceeds 0.8 year, an increase of the option vega could not compensate for the increase in the bias amount. Hence, the implied volatility appears to increase over this range of time to maturity.

Figure 9. Implied Volatility Across Time-to-Maturity: Case II Perfect Discrimination

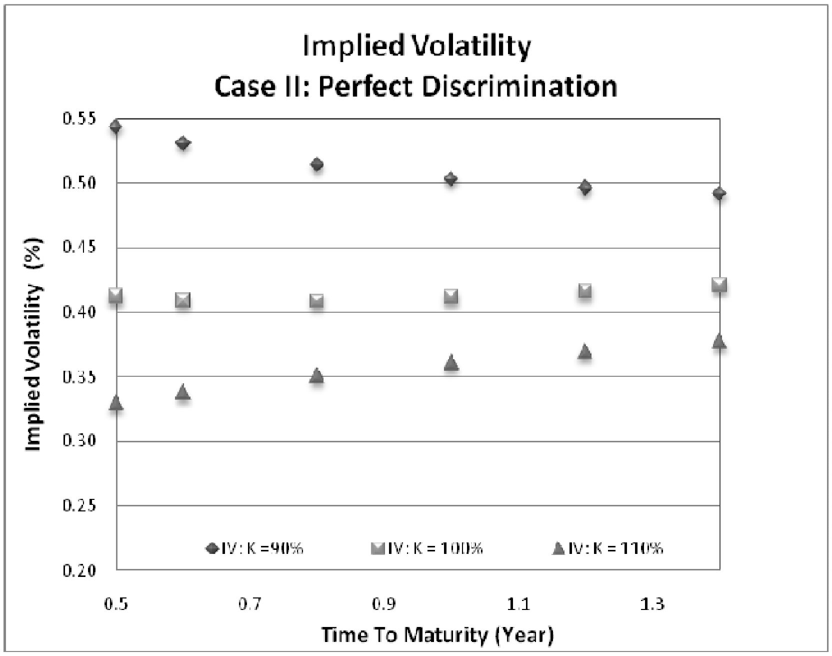
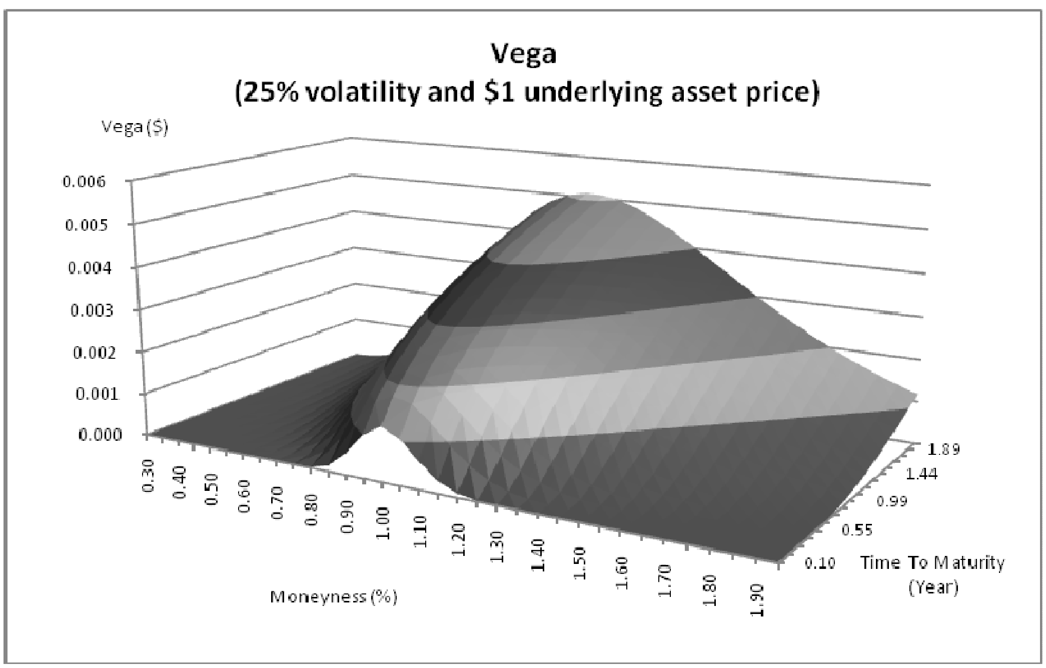


Figure 10. Vega (using 25% volatility and $1 Underlying Asset Price)



CHAPTER 4. EMPIRICAL RESULTS

4.1 Testing Hypotheses

The theoretical model presented in Chapter 3 suggests several testable hypotheses. In this chapter, several hypotheses are developed in order to apply to actual market data. Before proceeding to explain the determinants of the bias, I first examine the existence and the direction of the bias using the following testing hypothesis suggested by the theoretical model:

H1. Implied volatility is a positive biased estimator of future realized volatility.

To examine the H1 hypothesis, I construct a variable called "bias", which is the difference between the annualized implied volatility and the annualized realized volatility. The bias is measured as an annualized percentage. For example, if the annualized implied volatility equals 25% and the annualized realized volatility equals 15%, the annualized bias equals 10% (25%-10%). Details describing the calculations of the implied volatility and the realized volatility will be provided later in this chapter.

Positive (negative) bias means that the implied volatility over-(under-) estimates the realized volatility. The simplest method to determine the size of the bias is to calculate the mean of the bias across all observations. Another method is to regress the bias on a constant. The coefficient for the constant (or the intercept) produced by the regression is equal to the mean of the bias. The regression method is preferred because it allows us to test whether the calculated mean (or the regression coefficient) is statistically different than zero. If the mean/coefficient is statistically greater than zero, we fail to reject the H1 hypothesis, i.e., the implied volatility is a positive biased estimator of the realized volatility. In addition, the regression approach also allows us to adjust the standard errors using standard clustered-data errors described later in this chapter. Given the data used in this study, ignoring the clustered nature of the data will deflate the standard error. This could result in over-accepting the H1 hypothesis.

Define $S_{t,T}$ to be the bias, the difference between the implied volatility and the realized volatility. The implied volatility ($IV_{t,T}$) is the volatility of the option with T - t days to maturity. The future realized volatility ($RV_{t,T}$) is the realized volatility of the underlying asset during the current time *t* and the time to maturity *T*. To test the H1 hypothesis, the regression equation can be written as

$$S_{t,T} = \gamma + \varepsilon_{t,T} \tag{4.1.1}$$

where γ is the regression intercept. The magnitude and direction of the bias can be determined by the size and the sign of γ, respectively. If γ is statistically greater than zero, the implied volatility overestimates the future realized volatility, hence, we fail to reject the H1 hypothesis. Note that, in contrast to the hypothesis discussed in Section 2.2, setting up a hypothesis in this manner allows us to distinctively determine the direction of the bias.

The regression models examining the bias determinants are presented in equation (4.1.2) and (4.1.3). The theoretical model in (3.3.2) suggests that the bias embedded in the implied volatility is determined by the following variables: risk aversion coefficient (a), the option strike (k), the futures price (F), the futures volatility (σ), the risk free rate (r_f),the physical quantity (Q), the cost of trading (c_t), and time until maturity of the options (T). Unfortunately, the risk aversion coefficient is the only variable that is suggested by the theoretical model, and it cannot be observed from the market, at least from our data. This is because the data does not contain information that can be linked back to the risk aversion coefficient. However, by assuming that the risk aversion coefficient depends on individual preferences, the risk aversion coefficient is thus uncorrelated with other variables included in the model. Hence, the omission of this variable will not bias the coefficients estimated for other variables because the fundamental assumption of the linear regression that the error terms are uncorrelated with explanatory variables still holds. In addition, the cost of trading (c_t) is also not available in the data set. However, dummy variables representing commodity exchanges are used instead to represent different cost structure of each commodity exchange[24]. Hence, except for the risk aversion coefficient, these variables are included as explanatory variables in the regression models in (4.1.2) and (4.1.3).

Similar to the H1 hypothesis, several hypotheses can be constructed using these variables. However, only the following hypotheses are developed due to the importance of strikes and times to maturity on the bias as they are widely discussed in the literature. Since the numerical analysis presented in Chapter 3 suggests that the bias will vary simultaneously with respect to strikes and

[24] Details about the dummy variables representing commodity exchanges are discussed at the end of this section.

times to maturity, across different strikes, the model bias appears to have a smile shape and a downward-sloping shape in the non-discrimination case and the perfect-discrimination case, respectively. The numerical results further suggest that the bias can take different shapes across times to maturity depending on the market situation. The following hypotheses test these numerical findings:

H2. The implied volatility bias is non-constant across strikes.

H3. The implied volatility bias is non-constant across times to maturity.

To capture the volatility smile and the downward sloping shape across the option strikes, the option strikes are transformed into $\log\left(\frac{F}{k}\right)^2$ and $\log\left(\frac{F}{k}\right)$, respectively. By construction, $\log\left(\frac{F}{k}\right)^2$ has a U-shape and $\log\left(\frac{F}{k}\right)$ has a downward sloping shape across strikes. Hence, positive coefficients of these variables support the evidence of the smile and the downward slope of the biases across strikes. Moreover, to capture the curvature of the time to maturity, the square of time to maturity is also included in the regression model.

Moreover, the numerical results in Section 3.4 also suggest the impacts of these variables on the bias. The increase in future price and physical quantity should lead to an increase in the bias. For the no discrimination case, the increase in volatility should lead to a decrease in the bias and for the perfect discrimination case, the opposite result is found. The risk free rate appears to have very small impact on the bias. Note that, although, according to Section 3.4, the bias is defined in terms of the dollar amount and the bias discussed here is defined in terms of the annualized percentage, the signs suggested by the numerical results should be the same regardless of this unit difference.

Finally, the completeness of the data allows us to extend the regression model in (4.1.2) and (4.1.3) to test the following hypotheses:

H4. The implied volatility bias is different between puts and calls.

H5. The implied volatility bias is different over time.

H6. The implied volatility bias is different across the Exchanges.

To test the difference between puts and calls in the H4 hypothesis, the dummy variable *cp* is included where *cp* =1 represents call options and *cp* =0 represents put options. In addition, to test whether the bias differs across time, dummy variables for different years are included. Finally, to test whether the bias level varies across the commodity Exchanges, the trading cost variable (c_t) is replaced by dummy variables to represent each Exchange. These dummy variables allow us to compare trading costs across Exchanges. The numerical result suggests that the higher the trading cost, the higher the bias, i.e., we should see that the bias is higher for an exchange that has a higher trading cost. The empirical regression model for each futures market can be written as:

$$\begin{aligned} S_{t,T} &= \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} \\ &\quad + \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \varepsilon_{t,T} \end{aligned} \tag{4.1.2}$$

and the aggregate model used to test the difference in trading cost is:

$$\begin{aligned} S_{t,T} &= \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} \\ &\quad + \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \sum_{m \in Exchanges} \lambda_m Dexch_m + \varepsilon_{t,T} \end{aligned} \tag{4.1.3}$$

where Y_{90} is a dummy variable for year 1990, Y_{91} is a dummy variable for year 1991, etc., , and $Dexch_m$ is a dummy variable for exchange *m* where *m* represents an exchange included in the data.

4.2 Data and Methodology

Futures and options data were obtained from the Commodity Research Bureau (CRB). The data set is comprised of daily futures and options information, including high and low prices, closing price, volume, open interest, and option strikes. Although there is an increasing availability of high frequency intraday data, this study utilizes daily data rather than intraday data because the benefits resulting from using higher frequency data do not seem to outweigh the marginal cost of dealing with the massive data set (Neely (2004)), particularly when analyzing large numbers of commodity markets, as I do in this study

Table 5. Commodities and Exchanges

Ticker	Commodity	Exchange	Contract Months*	Period
Agricultural				
C-	Corn / No. 2 Yellow	CBOT	H,K,N,U,Z	1990-2008
CT	Cotton/1-1/16"	NYCE	H,K,N,V,Z	1990-2008
MW	Wheat/ No. 2 Soft Red	CBOT	H,K,N,U,Z	1991-2008
O-	Oats/ No. 2 White Heavy	CBOT	H,K,N,U,Z	1990-2008
RR	Rough Rice #2	CBOT	F,H,K,N,U,X	1992-2008
S-	Soybeans/ No. 1 Yellow	CBOT	F,H,K,N,Q,U,X	1990-2008
SM	Soybean Meal/ 48% Protein	CBOT	F,H,K,N,Q,U,V,Z	1990-2008
W-	Wheat/ No. 2 Soft Red	CBOT	H,K,N,U,Z	1990-2007
WA	Barley, Western / No. 1	WCE	H,K,N,V,Z	1997-2008
WF	Flaxseed / No. 1	WCE	F,H,K,N,U,V,X,Z	1993-2004
LB	Lumber/ Spruse-Pine Fir 2x4	CME	F,H,K,N,U,X	1990-2008
Soft				
CC	Cocoa/Ivory Coast	CSCE	H,K,N,U,Z	1990-2007
DE	Milk, BFP	CME	F,G,H,J,K,M,N,Q,U,V,X,Z	1998-2008
JO	Orange Juice, Frozen Concentrate	NYCE	F,H,K,N,U,X	1990-2008
KC	Coffee 'C' / Columbian	CSCE	H,K,N,U,Z	1990-2008
LW	Sugar #7/ White	LCE	H,K,Q,V,Z	1995-2008
SB	Sugar #11/ World Raw	CSCE	F,H,K,N,V	1990-2008
Livestock				
FC	Feeder Cattle/ Average	CME	F,H,J,K,Q,U,V,X	1990-2008
LC	Live Cattle/ Choice Average	CME	G,J,M,Q,V,Z	1990-2008
Precious Metal				
GC	Gold	COMEX	G,J,M,Q,V,Z	1990-2008
HG	Copper Hig Grade/ Scrap No. 2 Wire	COMEX	F,G,H,J,K,M,N,Q,U,V,X,Z	1990-2008
PL	Platinum	NYMEX	F,J,N,V	1991-2006
Energy				
BO	Soybean Oil/ Crude	CBOT	F,H,K,N,Q,U,V,Z	1990-2008
CL	Clude Oil	NYMEX	F,G,H,J,K,M,N,Q,U,V,X,Z	1990-2008
NG	Natural Gas	NYMEX	F,G,H,J,K,M,N,Q,U,V,X,Z	1992-2008
HO	Heating Oil #2	NYMEX	F,G,H,J,K,M,N,Q,U,V,X,Z	1990-2008

Note: Options' contract months represent the expiration months of the options. Contract months follow the standard exchange symbols below:

F = January	H = March	K = May	N = July	U = September	X = November
G = February	J = April	M = June	Q = August	V = October	Z = December

All commodities analyzed are listed in Table 5. These commodities are traded at various major exchanges, including the Chicago Board of Trade (CBOT), the Chicago Mercantile Exchange

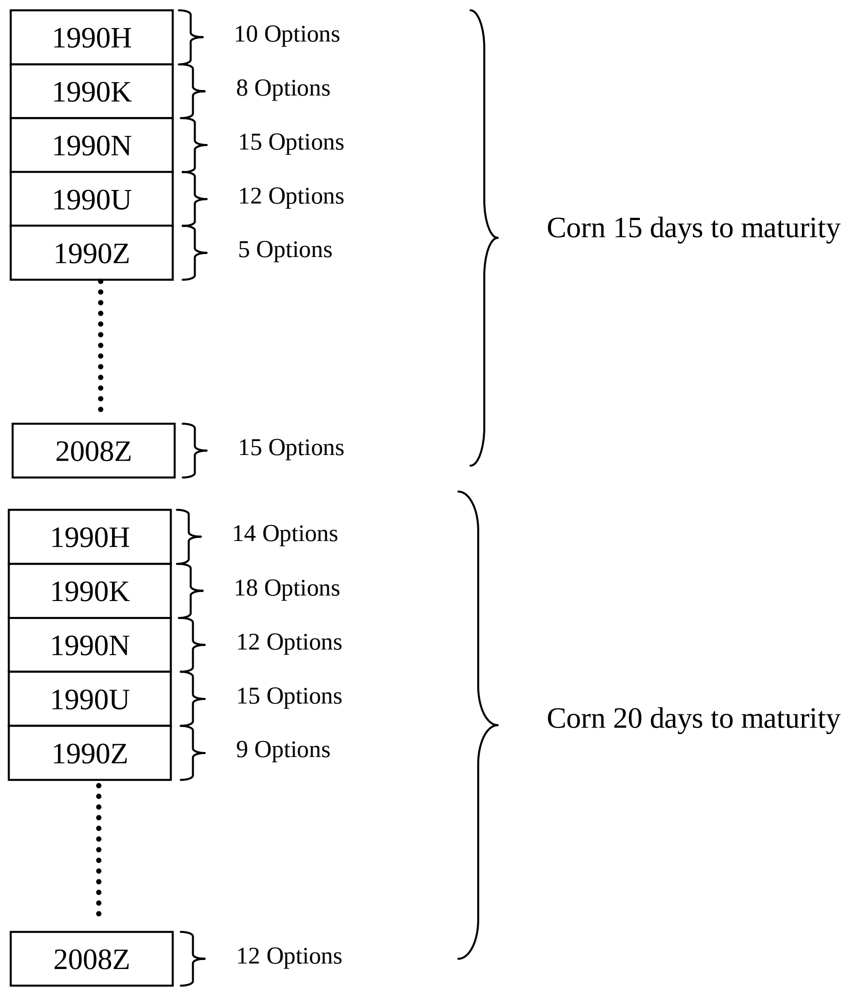
(CME), the Commodity Exchange, Inc (COMEX), the New York Mercantile Exchange (NYMEX), the New York Cotton Exchange (NYCE), the Minneapolis Grain Exchange (MGE), the LIFFE Commodity Exchange (LCE), the Coffee, Sugar & Cocoa Exchange (CSCE), and the Winnipeg Commodity Exchange (WCE). When available, the data began in 1990 and ended in 2008, although some commodities did not start options trading until later or the data are not available through the data vendor until 2008. In those cases only available data are included in the analysis.

The following section discusses the data construction method which can be best explained through an example. Consider the corn market which contains data between 1990 and 2008, all options traded during this period are included. However, if all daily observations are included, this will lead to high correlation among observations and a drop of correlation when options switch to the new futures contract. Therefore, to reduce correlation among observations over time, instead of using daily option data, only options with 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60 days to maturity are included and the clustered standard deviation, discussed in the next section, is used instead of the usual standard deviation. The following diagram demonstrates the example of the data where each option represents each observation.

According to the above figure, number of days to maturity is determined as the number of trading days from the analyzed date to the last date when the options were traded. Since the volatility calculated between Friday and Monday does not add much information to the volatility level calculated between trading days, I follow the approach of Jorion (1995) of using the number of trading days to maturity instead of the number of calendar days to maturity. The days to maturity value starts at 15 days in order to avoid extreme price fluctuations during the last month of expiration and ends at 60 days, which should be long enough to have adequate liquidity in the option markets and to provide a reasonable estimation of the future realized volatility. If there are many strikes traded on one futures contract (e.g., the August 2006 contract of natural gas 2006 has more than 250 strikes) some of these strikes are chosen to represent a reasonable moneyness level[25].

[25] We are unable to accommodate all strikes traded mainly due to the restriction in the number of columns in Excel 2003. However, strikes have been carefully chosen with higher weight to those with the moneyness between 60% and 140% from the future prices.

Figure 11. The Example of Data Structure: Corn



The final data clean-up process is to eliminate any inactive options. As options become deep out-of-the-money (OTM) or deep-in-the money (ITM), they become inactive. The values of deep OTM options are reduced to the minimal tick value. For example, a deep OTM corn option will have a closing value of 0.125 cents, the minimum tick value for corn options. Although these options are almost worthless, the exchanges continue to provide their closing values for settlement purposes. I exclude these options from the data set because the implied volatility calculated using these options will be inappropriately large and could distort the analysis. To determine which options to use, I first search for the threshold strike, the first strike with the minimum option value. I then discard options with strikes that are higher than the threshold strikes (for calls) and options with strikes that are lower than the threshold strike (for puts). For deep ITM options, I discard options with time value equal to zero. Time value is the difference between the option price and the intrinsic value. Similarly to the deep OTM options, zero time value means the options are deep ITM and are generally inactive. Due to some limitations in the original data provided by CRB, I also had to discard options that appeared to be inconsistent with their peers.

Currently, most research focuses on analyzing the implied volatility of ATM options, and, in particular, calculating the implied volatility by using the average of the implied volatility values from two ATM put and two ATM call options. This is because ATM options are also more liquid and hence produce fewer pricing errors. Furthermore, Beckers (1981) also suggests that ATM options provide the best estimation of the subsequently-realized volatility. However, focusing on the ATM options does not explain volatility skew across strikes. Therefore, since my model suggests that the option strike (or the option moneyness[26]) is one of the determinants of the bias, I unconventionally include options with a range of moneyness between 50% and 150%. In addition, the definition of ATM options can be different across different studies. In this research, ATM options refer to options with the moneyness ranges between 99% and 101%.

The empirical analysis proceeds in two stages. First I look for evidence of bias and then apply the theoretical model to determine the sources of bias.

4.3 Clustered Data

Although the sampling procedure discussed earlier helps to alleviate the correlation issue among observations over time, options from the same futures with similar days to maturity still have higher correlation than those collected from different futures with different days to maturity. For example, options with 15 days to maturity from September 1990 corn will have higher correlation among themselves when compared with options from September 1990 with 50 days to maturity or options from other contracts. Although the coefficients estimated from correlated data are still unbiased, statistical inference will be incorrect because the standard error is too small, perhaps leading to over-rejecting the standard null hypothesis of the regression. I employ the clustered robust standard error to correct for bias in statistical inference from the data structure. The data is clustered by the combination of the futures contract and number of days to maturity. The clustered robust standard error is calculated as

[26] In general, moneyness refers to the ratio of strike divided by the futures price.

$$V_{cluster} = (X'X)^{-1} \sum_{j=1}^{m} u_j^{'} u_j (X'X)^{-1} \tag{4.3.1}$$

where

$$u_j = \sum_{m} e_i x_i \tag{4.3.2}$$

where e_i is the residual of the i^{th} observation, x_i is a row vector of explanatory variables including the intercept, and *m* is the total number of clusters that is in turn equal to the total number of futures contracts. This clustered robust standard error is quite similar to the White robust standard error except that the sum over each observation in White's formula is replaced by the sum over each cluster. In STATA, the option `cluster` of the `regress` command automatically replaces the normal standard error with the clustered robust standard error.

4.4 Variables

Most explanatory variables are self-descriptive and can be easily obtained. The risk-free rate (r_f) represents the 3-month treasury yield posted by the Department of the Treasury[27]. Variable $\log\left(\frac{F}{k}\right)$ and $\left[\log\left(\frac{F}{k}\right)\right]^2$ are the forms of the option strike (k) and the future price (F). By construction, positive coefficients of these variables represent a downward slope and the volatility smile across option strikes. The volatility (σ) of the futures is calculated using the annualized historical volatility (HV) over the past 20 days. This is

$$HV_t = \left(\sqrt{\frac{1}{20} \sum_{i=t}^{20} \left[\ln\left(\frac{F_i}{F_{i-1}}\right) - E\left(\ln\left(\frac{F_i}{F_{i-1}}\right)\right)\right]^2} \right) * \sqrt{252} * 100 \tag{4.4.1}$$

For our purpose, the futures market is assumed to be unbiased. Therefore the realized volatility is calculated by setting the mean of $\ln\left(\frac{F_i}{F_{i-1}}\right)$ from the above equation to be zero. The price

[27] http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml accessed May 1, 2009.

of the option (p_t^{opt}) is directly obtained from the option market price. For the quantity demanded (Q), since I do not have data for the quantity demanded for the options, I proxy the option demand by the open interest of the futures. The dummy variables for years and exchanges follow the usual convention. For example, the value is equal to 1 when the year of the observation equals the year of the dummy variable and 0 otherwise. The calculation of the bias (S) is discussed in detail below.

Bias and Implied Volatility Calculation

The bias of an option is defined as the difference between the future realized volatility of the underlying asset over the remaining life of the options and the option's implied volatility, i.e.,

$$Bias = Implied\ Volatility - Realized\ Volatility \tag{4.4.2}$$

The realized volatility (RV) is calculated as an annualized standard deviation of the daily future return over the remaining life of the option according to the following formula:

$$RV_t = \left(\sqrt{\frac{1}{T-t} \sum_{i=t}^{T} \left[\ln\left(\frac{F_i}{F_{i-1}}\right) - E\left(\ln\left(\frac{F_i}{F_{i-1}}\right)\right) \right]^2} \right) * \sqrt{252} * 100 \tag{4.4.3}$$

where *t* denotes each trading days to maturity value of the options and $E\left(\ln\left(\frac{F_i}{F_{i-1}}\right)\right)$ is the mean of $\ln\left(\frac{F_i}{F_{i-1}}\right)$. Similar to the calculation of the historical volatility, the futures market is assumed to be unbiased. Hence, the realized volatility is calculated by setting the mean of $\ln\left(\frac{F_i}{F_{i-1}}\right)$ to be zero. As a result, equation (4.4.3) is reduced to the square root of the sum of the squares of the return.

As suggested by Jorion (1995), to avoid the well-known the Friday-to-Monday variance effect in which the variance over the weekend is slightly higher than the daily variance, the annualized volatility is calculated using the square root of 252 trading days instead of 365 calendar days.

The calculation of the implied volatility is not quite straightforward because it cannot be directly observed from the market. The Black formula for calls and puts on futures are presented below,

$$Call = e^{-rT}\left[FN(d_1) - KN(d_2)\right] \quad , \quad Put = e^{-rT}\left[KN(-d_2) - FN(-d_1)\right] \tag{4.4.4}$$

Where

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad , \quad d2 = \frac{\ln\left(\frac{F}{K}\right) - \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

k is the strike price, F is the future price, r_f is the risk free rate, T is time to maturity, and σ is the volatility of the underlying asset. Since this formula is not invertible, once the market prices of the options are observed, the implied volatility must be calculated through an iterative process. In the end, the implied volatility is the volatility that equates the option market price to the BSM price, i.e.,

$$P_{optionBSM}(\sigma_{IV}) = P_{optionMK} \tag{4.4.5}$$

where $P_{optionMK}$ is the market price of the option and $P_{optionBSM}(\sigma_{IV})$ is the BSM option value with implied volatility (σ_{IV}) as an input. The iteration process used to search for the implied volatility must be done numerically. The most common means for finding implied volatility is through the Bisection and the Newton-Raphson methods (Figlewski (1997)). However, since the result from using both methods is very similar, I decided to use the Newton- Raphson method for calculating the implied volatility. In a very few cases where the NR method failed to converge, the bisection method was used instead. Table 6 shows the result yielded by both the Bisection and Newton-Raphson methods. Using various input parameters, these two methods yield essentially the same implied volatilities.

Table 6. Implied Volatility Calculation from Bisection Method & Newton-Raphson Method

						Implied Volatility	
					Price of Call	Bisection	Newton - Raphson
F	K	T	r	σ	Option	Method	Method
100	100	15	2.00%	30.00%	2.42	30.00%	30.00%
100	90	30	2.00%	24.00%	10.16	24.00%	24.00%
100	110	60	3.00%	15.75%	0.20	15.75%	15.75%
110	120	120	3.00%	14.00%	0.65	14.00%	14.00%

With most exchange-traded options priced using the BSM or a variation of it that could allow for early exercise rights, it is appropriate to calculate the implied volatility using the generalized BSM formula that accounts for the early exercise right and to use the Newton Rahpson or the Bisection methods mentioned above to calculate the implied volatility.

4.5 Summary Statistics

The summary statistics of the bias are shown in Table 7. The statistics summary is broken down into categories puts, calls and a combination of puts and calls. The left panel summarizes the bias for combined options. The middle and the right panels summarize bias for calls and puts respectively. Summary statistics include numbers of observations, mean, standard deviation, minimum, and maximum of the bias for each futures market. The table also includes statistics by group and statistics of all observations combined.

From the table, the average bias of all options combined is 5.4%. The average bias is higher for calls (6.1%) than for puts (4.7%). Soft commodities have highest bias (9.4%) and energy commodities have lowest bias (4.0%). Soft commodities also have highest standard deviation (14.2), lowest minimum of bias (-71.7) and highest maximum of bias (227.8). Livestock commodities have the lowest standard deviation of the bias (6.6), highest minimum of bias (-19.9), and lowest maximum of bias (60.3). Except for flaxseed, feeder cattle, life cattle, and crude oil, the average of bias for calls appears to be higher than the average of bias for puts.

Heating oil has the lowest bias (1.9%), followed by spring wheat (3.0%) and soybean meal (3.4%). Cocoa, coffee, and world sugar have highest biases at 14.7%, 12.5% and 10.2% respectively. Among call options, heating oil, life cattle, and spring wheat have lowest biases and cocoa, coffee, and world sugar have highest biases. However, among put options, heating oil, soybean meal, and

Table 7. Summary Statistics of the Bias (the mean is in annualized percentage)

	ALL					CALLS					PUTS				
Markets	**Obs.**	**Mean**	**Std**	**Min**	**Max**	**Obs.**	**Mean**	**Std**	**Min**	**Max**	**Obs.**	**Mean**	**Std**	**Min**	**Max**
Agricultural															
Corn / No. 2 Yellow	17,750	5.3	8.4	-35	90.9	9,772	6.1	8.8	-29.2	90.9	7,978	4.3	7.9	-35	69.5
Cotton/1-1/16"	30,393	3.5	7.9	-17.2	84.4	15,794	3.8	8.3	-17.1	84.4	14,599	3.2	7.3	-17.2	52.8
Wheat/ Spring 14% Protein	12,194	3	7.5	-23.1	66.8	7,499	3.4	7.7	-23.1	60.7	4,695	2.4	7.2	-21.2	66.8
Oats/ No. 2 White Heavy	9,572	6.1	13.3	-52.6	99.8	5,525	7.5	14.3	-52.6	99.8	4,047	4.3	11.6	-47.2	67.9
Rough Rice #2	15,802	6.6	9.5	-32.1	92.2	9,018	7.3	9.7	-32.1	92.2	6,784	5.6	9.3	-31	76.5
Soybeans/ No. 1 Yellow	28,540	7.4	11.6	-30.2	100.7	17,084	9.2	12.3	-30.2	100.7	11,456	4.8	10	-28.2	65.7
Soybean Meal/ 48% Protein	23,916	3.4	10.5	-38.5	178.9	13,585	4.5	11.9	-38.5	178.9	10,331	1.9	8.1	-26.9	81.4
Wheat/ No. 2 Soft Red	43,390	4.6	9.8	-22.4	87	25,200	5.3	10.4	-22.4	87	18,190	3.6	8.6	-18.3	61.1
Barley, Western / No. 1	5,746	5.5	7.3	-22	66.8	2,845	5.6	7.6	-22	66.8	2,901	5.5	6.9	-20.2	35.2
Flaxseed / No. 1	7,222	4	6.4	-22	45.6	3,778	3.7	6.6	-22	45.6	3,444	4.2	6.1	-11.1	34.6
Lumber/ Spruce-Pine Fir 2x4	19,258	5.7	7.1	-19	81.8	10,330	5.8	7.2	-18.2	81.8	8,928	5.5	7	-19	56.6
Average	**213,783**	**5**	**9.6**	**-52.6**	**178.9**	**120,430**	**5.8**	**10.3**	**-52.6**	**178.9**	**93,353**	**3.9**	**8.4**	**-47.2**	**81.4**
Soft															
Cocoa/Ivory Coast	4,800	14.7	19.2	-27.5	120.7	2,948	20.2	21.2	-26.3	120.7	1,852	5.8	10.5	-27.5	74.1
Milk, BFP	24,044	8	10.9	-32.7	101.1	12,750	8.6	11.5	-29.2	101.1	11,294	7.2	10.1	-32.7	73.5
Orange Juice, Frozen Concentrate	24,415	6	13.3	-71.7	92.7	11,628	8	14.5	-71.7	92.7	12,787	4.2	11.9	-71.6	69.4
Coffee 'C' / Columbian	28,857	12.5	15.8	-60.3	99.3	18,777	12.9	15.9	-60.2	99.3	10,080	11.9	15.4	-60.3	82.5
Sugar #7/ White	6,635	7.9	13.5	-28.3	112.2	3,678	8.4	15	-28.3	112.2	2,957	7.2	11.2	-28.2	63.6
Sugar #11/ World Raw	17,188	10.2	15.4	-25.3	227.8	8,986	12.2	18.3	-23.3	227.8	8,202	8.1	11	-25.3	151.2
Average	**105,939**	**9.4**	**14.5**	**-71.7**	**227.8**	**58,767**	**11**	**15.7**	**-71.7**	**227.8**	**47,172**	**7.5**	**12.4**	**-71.6**	**151.2**

Table 7. (continued)

	ALL					CALLS					PUTS				
Markets	**Obs.**	**Mean**	**Std**	**Min**	**Max**	**Obs.**	**Mean**	**Std**	**Min**	**Max**	**Obs.**	**Mean**	**Std**	**Min**	**Max**
Livestock															
Feeder Cattle/ Average	18,132	5.3	7.1	-19.9	60.3	8,250	4.2	5.6	-19.9	57.8	9,882	6.2	8.1	-16.9	60.3
Live Cattle/ Choice Average	16,317	4.2	5.9	-14.1	38.1	7,613	3.1	4.3	-12.2	33.6	8,704	5.1	6.9	-14.1	38.1
Average	**34,449**	**4.7**	**6.6**	**-19.9**	**60.3**	**15,863**	**3.7**	**5**	**-19.9**	**57.8**	**18,586**	**5.7**	**7.6**	**-16.9**	**60.3**
Precious Metal															
Gold	40,892	7.5	10	-31.4	82.3	19,659	7.7	10.4	-31.4	82.3	21,233	7.3	9.6	-31.4	69.6
Copper High Grade/ Scrap No. 2 Wire	60,822	6.3	13.9	-63.3	104.4	29,529	8.1	16	-58.4	104.4	31,293	4.6	11.2	-63.3	84.9
Platinum	7,967	3.7	7.3	-24.9	39.7	4,751	3.8	7.2	-24.9	35.7	3,216	3.6	7.6	-24.9	39.7
Average	**109,681**	**6.6**	**12.2**	**-63.3**	**104.4**	**53,939**	**7.6**	**13.6**	**-58.4**	**104.4**	**55,742**	**5.6**	**10.5**	**-63.3**	**84.9**
Energy															
Soybean Oil/ Crude	20,796	4.9	9.7	-33.1	107	11,821	5.7	10.9	-33.1	107	8,975	3.8	7.6	-23.2	64.7
Crude Oil	84,521	5	9.6	-51.9	130.8	47,854	4.8	9	-51.9	130.8	36,667	5.2	10.4	-51.4	97.2
Natural Gas	75,516	4.9	12.9	-48	110.8	39,555	5	12.7	-48	108.9	35,961	4.7	13.1	-46.7	110.8
Heating Oil #2	79,175	1.9	8.5	-57.3	76	42,732	2.7	8.4	-56.8	76	36,443	0.9	8.6	-57.3	66.5
Average	**260,008**	**4**	**10.5**	**-57.3**	**130.8**	**141,962**	**4.3**	**10.2**	**-56.8**	**130.8**	**118,046**	**3.6**	**10.8**	**-57.3**	**110.8**
ALL	**730,940**	**5.4**	**11.1**	**-71.7**	**227.8**	**394,540**	**6.1**	**11.8**	**-71.7**	**227.8**	**336,400**	**4.7**	**10.3**	**-71.6**	**151.2**

spring wheat have lowest biases and orange juice, world sugar, and gold appear to have highest biases.

4.6 Empirical Results

Empirical results are presented in two parts. The first part shows the result with respect to hypothesis H1; that is, to examine whether the implied volatility is a positive biased estimator of the future realized volatility. The second part shows the result with respect to the remaining hypotheses.

4.6.1 The implied volatility is a biased estimator of the future realized volatility

The result of whether the implied volatility is a biased estimator of the future realized volatility is shown below. Table 8 and Table 9 present the regression result for hypothesis H1 (equation (4.1.1). Due to higher liquidity, ATM options as opposed to non-ATM options are widely believed to have the smallest bias. Hence, most research focuses their analysis of the bias embedded in the options on ATM options. Following conventional methods, I separate options into two types: ATM options and non-ATM options.

Table 8 shows the regression result of the bias for ATM options. Positive bias means that the implied volatility overestimates the realized volatility and negative bias means that the implied volatility underestimates the realized volatility.

Examining Table 8, twenty out of twenty-six markets exhibit significant positive bias. Only the soybean meal market exhibits significant negative bias. The biases in cotton, oats, wheat, cocoa, orange juice and heating oil markets are not statistically different from zero. The overall bias is estimated to be 1.108% which means that, when considering all markets, on average the implied volatility over-estimates the realized volatility by 1.108%. ATM options on barley futures are shown to have the highest bias at 4.35%. Soybean oil options have the lowest positive bias at 0.666%

The regression of the bias for non-ATM options is shown in Table 9. All bias estimations in all markets are positive and statistically different from zero. The average of the biases in all market is 8.142%, approximately 7% higher than the estimate from ATM options, consistent with general belief.

Table 8. Regression Result: Existence of Bias for ATM options

Regression $S_{t,T} = \gamma + \varepsilon_{t,T}$ for ATM options where $S_{t,T}$ (bias) = implied volatility – realized volatility.

Market	Bias	Market	Bias	Market	Bias
All	**1.108*** **				
Agricultural		**Soft**		**Precious Metal**	
Corn / No. 2 Yellow	1.835***	Cocoa/Ivory Coast	-0.994	Gold	0.805***
				Copper High Grade/	
Cotton/1-1/16"	0.124	Milk, BFP	3.279***	Scrap No. 2 Wire	0.967***
Wheat/ Spring 14%		Orange Juice, Frozen			
Protein	0.714**	Concentrate	-0.598	Platinum	1.213**
Oats/ No. 2 White Heavy	0.799	Coffee 'C' / Columbian	3.330***		
Rough Rice #2	3.506***	Sugar #7/ White	1.505**		
Soybeans/ No. 1 Yellow	0.666**	Sugar #11/ World Raw	1.132*		
Soybean Meal/ 48%					
Protein	-0.602**			**Energy**	
Wheat/ No. 2 Soft Red	-0.159			Soybean Oil/ Crude	0.666***
Barley, Western / No. 1	4.350***	**Livestock**		Clude Oil	1.255***
Flaxseed / No. 1	3.511***	Feeder Cattle/ Average	1.458***	Natural Gas	2.686***
Lumber/ Spruse-Pine Fir		Live Cattle/ Choice			
2x4	3.266***	Average	0.996***	Heating Oil #2	0.118

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

However, one important characteristic of ATM options is that they have a higher cost of hedging than non-ATM options. Given the same time to maturity, ATM options have higher vega and gamma. Hence, the risk management of the ATM options becomes more involved. In many cases, the cost of managing risk for ATM options could exceed the savings resulting from more liquidity.

Table 9. Regression Result: Existence of Bias for Non-ATM options

Regression $S_{t,T} = \gamma + \varepsilon_{t,T}$ for non-ATM options where $S_{t,T}$ (bias) = implied volatility – realized volatility.

Market	Bias	Market	Bias	Market	Bias
All	**8.142*** **				
Agricultural		**Drinks**		**Precious metal**	
Corn / No. 2 Yellow	5.958***	Cocoa/Ivory Coast	18.77***	Gold	9.005***
Cotton/1-1/16"	4.462***	Milk, BFP	8.507***	Copper High Grade/ Scrap No. 2 Wire	7.727***
Wheat/ Spring 14% Protein	3.277***	Orange Juice, Frozen Concentrate	10.60***	Platinum	4.032***
Oats/ No. 2 White Heavy	7.726***	Coffee 'C' / Columbian	28.12***		
Rough Rice #2	7.462***	Sugar #7/ White	12.76***		
Soybeans/ No. 1 Yellow	8.951***	Sugar #11/ World Raw	17.94***		
Soybean Meal/ 48% Protein	5.750***			**Energy**	
Wheat/ No. 2 Soft Red	5.629***			Soybean Oil/ Crude	6.695***
Barley, Western / No. 1	5.692***	**Livestock**		Crude Oil	5.601***
Flaxseed / No. 1	4.009***	Feeder Cattle/ Average	6.073***	Natural Gas	7.519***
Lumber/ Spruce-Pine Fir 2x4	5.942***	Live Cattle/ Choice Average	4.756***	Heating Oil #2	2.183***

Note:
(c) t statistics in parenthesis
(d) * p< 0.05, ** p< 0.01, *** p< 0.0001

Therefore, although non-ATM options have significantly higher volatility bias, whether or not these options will have higher monetary bias is still questionable. To answer this question, I calculate the fair value of the option using the BSM option price (equation (4.4.4)) with the actual realized volatility over the remaining life of the option. The difference between the actual option price and the fair price is the bias in dollar terms. This is different than the definition of bias used in the empirical result (e.g. the definition of the bias used in Section 4.1) where the bias is defined as the percentage difference between the implied and realized volatility. The bias is then multiplied by contract size and, when applicable, divided by 100 to convert into dollar units.

However, one important characteristic of ATM options is that they have a higher cost of hedging than non-ATM options. Given the same time to maturity, ATM options have higher vega and gamma. Hence, the risk management of the ATM options becomes more involved. In many cases, the cost of managing risk for ATM options could exceed the savings resulting from more liquidity.

Table 10. Option Bias in Dollars

Market	Contract Size	Unit	Price Per Unit	Price Difference** = P(actual) - P(RV) ATM	Non-ATM	ALL	Average Bias Per Contract ($) ATM	Non-ATM	ALL
Agricultural									
Corn / No. 2 Yellow	5,000	bushel	cents/Bushel	0.828	0.550	0.568	41	28	28
Cotton/1-1/16"	50,000	pounds	cents/pound	0.026	0.031	0.031	13	15	15
Wheat/ Spring 14% Protein	5,000	bushel	cents/Bushel	0.319	0.381	0.375	16	19	19
Oats/ No. 2 White Heavy	5,000	bushel	Cents/Bushel	-0.108	0.185	0.171	-5	9	9
Rough Rice #2	2,000	cwt	dollars/cwt	0.046	0.033	0.034	92	66	68
Soybeans/ No. 1 Yellow	5,000	bushel	cents/Bushel	0.869	0.767	0.773	43	38	39
Soybean Meal/ 48% Protein	100	tons	dollars/ton	-0.245	0.000	-0.018	-25	0	-2
Wheat/ No. 2 Soft Red	5,000	bushel	cents/Bushel	-0.094	0.260	0.240	-5	13	12
Barley, Western / No. 1	100	tons	CAD/ton	0.927	0.403	0.443	93*	40*	44*
Flaxseed / No. 1	100	tons	CAD/ton	1.648	0.732	0.814	127*	56*	81*
Lumber/ Spruce-Pine Fir 2x4	80,000	board feet	dollars/1000 bf	1.574	1.213	1.238	97	75	99
Drinks									
Cocoa/Ivory Coast	10	metric ton	dollars/ton	-1.303	1.060	1.020	-13	11	10
Milk, BFP	50,000	pounds	cents/pound	0.083	0.031	0.035	41	15	17
Orange Juice, Frozen Concentrate	15,000	pound	cents/pound	-0.244	0.120	0.107	-37	18	16
Coffee 'C' / Columbian	37,500	pound	cents/pound	0.544	0.445	0.447	204	167	168
Sugar #7/ White	50	metric ton	dollars/ton	0.249	0.320	0.316	12	16	16
Sugar #11/ World Raw	112,000	pounds	cents/pound	0.017	0.022	0.022	19	25	25
Livestock									
Feeder Cattle/ Average	50,000	pounds	cents/pound	0.189	0.158	0.163	94	79	82
Live Cattle/ Choice Average	40,000	pounds	cents/pound	0.118	0.109	0.110	47	44	44
Precious metal									
Gold	100	ounces	dollars/ounces	0.135	0.270	0.261	13	27	26
Copper High Grade/ Scrap No. 2 Wire	25,000	pounds	cents/pound	0.210	0.171	0.173	53	43	43
Platinum	50	ounces	dollars/ounces	0.688	0.717	0.714	34	36	36
Energy									
Soybean Oil/ Crude	60,000	pounds	cent/pound	0.017	0.034	0.033	10	20	20
Crude Oil	1,000	Barrels	dollars/Barrel	0.047	0.048	0.048	47	48	48
Natural Gas	10,000	MMBtu	dollars/MMBtu	0.025	0.019	0.020	247	194	196
Heating Oil #2	42,000	gallon	dollars/gallon	0.001	0.001	0.001	42	52	51

*Contract specified in Canadian Dollars. Convert into U.S. dollars using exchange rate 1.3 Canadian dollar per1 U.S. dollar.
** Price Difference could be in cents or dollars depending on the price per unit.

Therefore, although non-ATM options have significantly higher volatility bias, whether or not these options will have higher monetary bias is still questionable. To answer this question, I calculate the fair value of the option using the BSM option price (equation (4.4.4)) with the actual realized volatility over the remaining life of the option. The difference between the actual option price and the fair price is the bias in dollar terms. This is different than the definition of bias used in the empirical result (e.g. the definition of the bias used in Section 4.1) where the bias is defined as the percentage difference between the implied and realized volatility. The bias is then multiplied by contract size and, when applicable, divided by 100 to convert into dollar units.

However, one important characteristic of ATM options is that they have a higher cost of hedging than non-ATM options. Given the same time to maturity, ATM options have higher vega and gamma. Hence, the risk management of the ATM options becomes more involved. In many cases, the cost of managing risk for ATM options could exceed the savings resulting from more liquidity.

Table 10 shows the result of this calculation, averaged by market. Negative bias means that the actual option prices are lower than the fair option price. Similar to the result from Table 8, for ATM options, markets with negative bias are orange juice, soybean meal, cocoa, oats, and wheat, respectively. Orange juice has the highest negative bias in dollars ($37/contract). Although the percentage bias for oats is positive, the dollar bias is slightly negative ($5/contract). This negative is mainly driven by large negative bias in 2008, the year when the financial crisis started. It is possible for the market to have positive average percentage bias and negative monetary bias because option values are non-linear functions of volatility.

As we might expect, in all markets, the dollar bias is positive for non-ATM options. When both types of options are combined, the bias is found to be positive in all markets except for the soybean meal market. The slight negative dollar bias ($-2/contract) in the soybean meal market is primarily driven by options traded during 2004 and 2008. Natural gas ($196) and coffee ($168) are the only two markets that have bias exceeding $100/contract. For other markets, the bias ranges from $9/contract in oats to $99/contract in lumber.

Notice that the ATM dollar bias is in the same range as commission charges. If we view access to a seat as a barrier to entry for option writers then this result makes sense. Those who have access to a seat must compete with those writers who do not. If the seat holders attempt to charge a bias that is greater than the barrier to entry then they will attract competition from writers who do not have a seat. The equilibrium ATM bias appears to be the transactions cost faced by the off exchange option writers.

The empirical results in Table 8, Table 9, and Table 10 support the hypothesis H1. That is, the implied volatility is, in twenty-five markets, a positive biased estimator of the realized volatility for ATM options and, for all markets, a positive biased estimator of the realized volatility for non-ATM options. Hence, over all, we fail to reject the H1 hypothesis that is the implied volatility is positive biased estimator of the realized volatility.

One could raise a concern that the bias shown here results from the bid-ask spread in the option market. However, the data was collected on a daily basis using the closing price which, according to the Chicago Mercantile Exchange (CME) staff, represents the mid price[28]. Therefore, the estimation should not be contaminated by the effect of the bid-ask spread.

4.6.2 Determinants of the implied volatility bias

The results for the remaining hypotheses are presented in this section. Table 11shows the regression result for agricultural commodity markets. Table 12 shows the result for soft commodity and livestock markets, Table 13 shows the result for the precious metal and energy markets, and, Table 14 shows the result for all markets combined. The theoretical model seems to fit the data quite well, with only two markets having less than 30% R^2, eleven markets having between 30%-40% R^2, and thirteen markets having more than 50% R^2.

The plots of the bias surface for each market are included in Appendix A. These plots are generated by taking the average of the volatility bias since the data in the dataset became available, in many cases, between 1990 and 2008. The bias surfaces are then generated by having the days to maturity and the moneyness (%) on the x-axes and the bias (%) on the y-axis. As before, the moneyness (%) equals the ratio of the strike to the future price.

The result from all market fails to reject the H2 hypothesis that the implied volatility bias is non-constant across strikes. The variable represents the bias across strikes, either $\log\left(\frac{F}{k}\right)$ or $\log\left(\frac{F}{k}\right)^2$ or both are strongly statistically significant in all markets. Positive value of $\log\left(\frac{F}{k}\right)^2$ reflect the smile shape of the bias. Appendix A shows the bias surface. Although not all markets show a perfectly symmetric smile shape, some degree of the smile can be found in all markets, particularly when options get closer to maturity date. Cotton, barley, feeder cattle, live cattle, copper, and crude

[28] According to the phone conversation with Tom Lord, Director of Settlements at 312.341.3116, the closing price generated by the CME should represent the mid price between the bid-ask spread.

oil have positive and significant coefficients for the $\log\left(\frac{F}{k}\right)$ variable; that is, the volatility bias in these markets is downward sloping shape across strikes. The bias surface plots in Appendix A show the pattern of downward sloping across the moneyness axis among these markets. The downward sloping shape is more prominent when options have more days to maturity.

Eighteen out of twenty-six markets have negative coefficients for the *Days to maturity* variable with a significant level of at least 5%. Hence, we fail to reject the H3 hypothesis that the implied volatility bias is non-constant across times to maturity. According to the coefficient of the *Days to maturity,* 5 days closer to maturity will increase the bias by approximately 1% (e.g. in cotton and oats markets) to 5% (in cocoa market). Markets that reject the H3 hypothesis include corn, spring wheat, soybean meal, flaxseed, lumber, life cattle, platinum, and heating oil. In these markets, the difference between the bias at 15 days to maturity and the bias at 60 days to maturity appears to be smaller than in other markets. This is especially true for 70% moneyness where the difference between the bias at 15 days to maturity and 60 days to maturity for these markets is less than 15%, significantly less that other markets in which the value is greater than approximately 20%. The variable *Days to maturity^2* captures the rate of impact of the variable *Days to maturity*. When *Days to maturity^2* is significant, it has a positive sign, meaning that the impact of the number of days to maturity on the bias is increasing at an increasing rate.

The bias difference between puts and calls (the H4 hypothesis) is tested using the variable *cp*. The value of *cp* equals 1 for calls and 0 for puts. In eighteen markets, the coefficient of *cp* is positive and significant. This means that the bias is higher in calls than in puts. The coefficient ranges from 0.3% in the gold market to 1.5% in the world sugar market. The negative coefficient of the *cp* variable can be found in two markets: spring wheat and cocoa. This means that, in these markets, the bias is higher in puts than in calls. When considering the coefficient of the *cp* and the $\log\left(\frac{F}{k}\right)$ variables together, one interesting point can be made. In feeder cattle, life cattle, and barley markets, the coefficient of *cp* is insignificant but the coefficient of $\log\left(\frac{F}{k}\right)$ is strongly significant. The insignificant coefficient for the *cp* variable implies that in these markets, call options are not more expensive than put options. When market structure is considered, particularly for the feeder cattle and the life cattle markets, this is true because these markets are very sensitive to animal diseases. Hence, more protection is directed toward protecting the risk of price decrease. This market nature could bid up the price for lower strikes or the prices of put options. As a result, the prices for call options in these markets do not show more bias that that of put options. The bias surfaces in Appendix A also tell the same story. The surfaces for these markets clearly exhibit downward slope across moneyness. In conclusion, since most markets show strongly significant coefficients, I fail to reject the H4 hypothesis that the implied volatility bias is different between puts and calls.

The H5 hypothesis can be tested using a set of dummy variables in which each variable represents a year of data. The dummy variable has a value of 1 if the data is collected during that year and 0 otherwise. If there is no data during any particular year, the coefficient of that year's dummy variable is left blank. Due to the shift of the trading system from the open outcry system to the electronic system starting in 2001, the year 2001 is used as the base year. As shown in the tables below, coefficients of year dummy variables have both positive and negative values depending on markets and years. For example, coefficients of the year 2002 to 2008 are negative and statistically significant for the sugar #7 market. This means that, when comparing with the year 2001, the bias in this market was smaller during this time period. A completely opposite result is found in the natural gas market where coefficients during the same period are positive and significant. However, with more than half of the coefficients statistically different from zero, we fail to reject the H5 hypothesis that the bias is non-constant over time.

To test the H6 hypothesis, whether the implied volatility bias differs across different exchanges, I combined the data from all markets into the regression shown in Table 14. Each observation represents one option strike traded on a particular exchange. Therefore, the exchange dummy variables take values of 1 if the option was traded on that exchange and 0 otherwise. Using the Chicago Board of Trade (CBOT) as a base case, Table 14 shows that the bias from the New York Mercantile Exchange (NYMEX), the Minneapolis Grain Exchange (MGE), and the New York Cotton Exchange (NYCE) and are -1.46% ,-1.11%, and -0.91% lower than the bias from the CBOT. When comparing across Exchanges, The Chicago Mercantile Exchange (CME) appears to have the highest level of bias. With all coefficients statistically different from zero, we clearly fail to reject hypothesis H6.

As discussed in Section 4.1, we next consider other variables included in the regression models (equation (4.1.2) and (4.1.3)). The physical quantity of the underlying asset (Q) is proxied by the *Open Interest* variable, which is the open interest of the underlying future market (per 100,000 contracts). The numerical analysis suggests that the decrease in physical quantity would lead to the decrease in the bias. However, in most markets, this variable does not appear to have any impact on the bias level. Only ten markets show significant coefficient of this variable. Of these ten markets, five markets show positive signs and another five markets show negative signs. This mixed result could happen because, although the decrease in physical quantity could lead to smaller demand for hedge, hence, smaller bias, the decrease in physical quantity could also lead to less liquidity which could drive up the demand for hedging in order to protect against price fluctuation, hence, larger bias. Therefore, with these two forces working against each other, the impact of the *Open interest* variable is different depending on the market and cannot be conclusively determined.

In most markets, the *Historical return (20 days)* variable proxied for the futures price (F) does not seem to have much impact on the bias level in contrast to the *Historical volatility (20 days)* variable proxied for the futures volatility (σ). The coefficient of the *Historical return (20 days)* variable is only significant among energy commodities where the coefficients are positive and the world sugar market where the coefficient is negative. The volatility in the futures market seems to have a significant impact on the bias in all but four markets: corn, flaxseed, cocoa, and milk. Similar to the sign expected from the perfect discrimination case, in most markets, the higher the volatility in the futures market, the higher the bias in the options market. Intuitively, this is because, as the underlying market becomes more volatile, the market is more willing to pay more bias in order to hedge against the increased uncertainty.

Although the numerical results show that the impact of the risk free rate is very small in terms of monetary bias, the impact of the *Risk free rate* (r_f) in terms of annualized percentage bias is statistically greater than zero in sixteen markets. The 1% increase of the risk free rate could result in 0-3% increase of the bias. Positive coefficient of the risk free rate means that as the cost of borrowing and the option price increases, the bias also increases. Finally, the impact of the *Option price* variables (P_{opt}) are mostly positive which means that ITM options will have higher volatility bias than other options.

Table 11. Regression Result: Agricultural Commodities

$$S_{t,T} = \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \varepsilon_{t,T}$$

Variables	Corn / No. 2 Yellow	Cotton/1-1/16"	Wheat/ Spring 14% Protein	Oats/ No. 2 White Heavy	Rough Rice #2
Open Interest	-1.360***	2.817***	1.89	24.18**	-20.99
Log(s/k)	-18.11***	1.651***	-16.63***	-7.601***	-8.929***
Log(s/k)^2	83.06***	108.9***	115.4***	135.9***	109.0***
Historical volatility (20 days)	-0.0114	0.226***	0.0765*	0.187***	0.167***
Historical return (20 days)	28.10***	-10.19	8.106	8.111	14.26
Risk free rate	2.137***	1.318***	1.379***	3.938***	1.386**
Option price	0.0136	0.171***	0.0383***	0.0669***	0.502***
Days to maturity	-0.126	-0.307***	-0.0448	-0.383**	-0.219*
Days to maturity ^2	0.000142	0.00265**	0.000145	0.00407*	0.00208
Calls / Puts	0.449**	0.710***	-0.278*	0.385*	0.897***
Year 90	-8.989***	4.118**		-15.09***	
Year 91	-8.240***	3.205**	-4.315**	-7.013**	
Year 92	-0.617	5.575***	-1.115	0.471	10.06***
Year 93	0.461	7.478***	-3.908***	5.396**	-1.553
Year 94	-2.302*	4.813***	-2.006***	2.665	2.329
Year 95	-3.685**	2.735*	-4.179**	-4.028	4.730**
Year 96	-7.565***	5.571***	-7.676***	-12.26***	5.165***
Year 97	-4.659***	9.386***	-4.464***	-4.56	9.471***
Year 98	-3.900***	3.051*	-1.163	2.139	6.215***
Year 99	-4.211***	5.551***	-4.734***	5.051*	0.759
Year 00	-3.484*	1.547	0.0665	-0.519	3.956*
Year 02	2.638	5.607***	-6.244***	-0.712	7.486***
Year 03	1.179	5.267**	-0.123	8.693***	9.609***
Year 04	3.675	-0.35	-1.512	6.834**	6.848**
Year 05	1.804	4.763***	-0.969	2.994	8.477***
Year 06	-3.944*	7.723***	-6.414***	-5.653**	4.038*
Year 07	-3.249	5.059***	-6.513***	-4.823*	4.764**
Year 08	-0.736	1.65	-12.73***	-6.896**	2.867
Intercept	3.846	-9.607***	-1.861	-12.63**	-6.275
Obs.	17,750	30,393	12,194	9,572	15,802
R^2	0.424	0.557	0.421	0.522	0.434

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

Table 11. (continued)

$$S_{t,T} = \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma^2_{-20} + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \varepsilon_{t,T}$$

Variables	Soybeans/ No. 1 Yellow	Soybean Meal/ 48% Protein	Wheat/ No. 2 Soft Red	Barley, Western / No. 1	Flaxseed / No. 1	Lumber/ Spruce-Pine Fir 2x4
Open Interest	1.018***	-1.431	-0.54	34.44*	40.61	-4.267
Log(s/k)	-13.00***	-15.44***	-9.557***	4.521***	-0.574	1.57
Log(s/k)^2	138.5***	132.2***	133.9***	108.2***	153.7***	74.54***
Historical volatility (20 days)	0.0900***	0.175***	0.146***	-0.343***	-0.0593	0.303***
Historical return (20 days)	2.985	26.77***	2.749	-6.412	2.771	9.868
Risk free rate	1.596***	2.880***	0.796*	-0.505	1.209**	0.438
Option price	0.0368***	0.0165**	0.0708***	0.0506***	0.00536	0.0367***
Days to maturity	-0.342***	-0.143	-0.368***	-0.345**	-0.2	-0.102
Days to maturity ^2	0.00271**	0.00051	0.00334***	0.00340*	0.0017	0.000941
Calls / Puts	1.443***	0.199	0.373***	-0.115	-0.315	0.663**
Year 90	-4.145***	-13.57***	-4.211**			3.62
Year 91	-5.151***	-10.23***	-3.494**			4.102*
Year 92	4.514***	3.018***	0.437			7.064***
Year 93	3.780***	1.902*	0.26		-1.749	2.239
Year 94	0.188	0.76	1.104		2.046	5.046***
Year 95	0.59	-5.481***	-3.311**		-3.796***	-2.368
Year 96	-1.439*	-3.003***	-7.299***		-0.392	0.758
Year 97	-5.085***	-8.560***	-3.439***	-2.541*	6.615*	4.757***
Year 98	0.304	-5.830***	-0.626	-1.016	-1.726	2.459
Year 99	-1.011	-6.434***	-2.020*	-0.175	-1.545	1.472
Year 00	3.708***	-2.248*	1.651	-2.478	-2.074	4.852**
Year 02	2.650*	4.673***	-2.552*	-0.473	9.092***	3.834*
Year 03	2.764*	5.314***	-5.465***	-5.965*	8.190***	5.686**
Year 04	-7.139***	-4.194*	-5.212***	-2.88	19.60***	5.624**
Year 05	-3.104**	0.515	0.779	2.408		3.169*
Year 06	-1.642*	-0.636	-3.197**	-4.364**		2.917*
Year 07	-5.195***	-7.446***	-7.172***	-0.347		3.939**
Year 08	0.186	-3.728**		-6.974**		3.554
Intercept	-0.0834	-8.581***	2.898	17.78***	1.181	-7.733*
Obs.	28,540	23,916	43,390	5,746	7,222	19,258
R^2	0.652	0.496	0.633	0.347	0.318	0.319

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

Table 12. Regression Result: Soft Commodities & Livestock

$$S_{t,T} = \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} + \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \varepsilon_{t,T}$$

	Soft Commodities						Lifestock	
Variables	Cocoa/Ivory Coast	Milk, BFP	Orange Juice, Frozen Concentrate	Coffee 'C' / Columbian	Sugar #7/ White	Sugar #11/ World Raw	Feeder Cattle/ Average	Live Cattle/ Choice Average
Open Interest	1.342	-34.08	9.206	-5.859*	2.130	-1.301*	-9.127*	-0.679
Log(s/k)	0.797	-5.396***	-4.055***	-18.76***	-0.944	0.947	28.76***	21.25***
Log(s/k)^2	101.7***	143.8***	132.0***	104.2***	129.7***	117.2***	153.9***	128.3***
Historical volatility (20 days)	-0.0277	-0.00665	0.112**	0.150***	0.152**	0.276***	0.186***	0.270***
Historical return (20 days)	-22.75	-7.716	15.41	-15.74	-13.67	-29.11***	-1.387	0.231
Risk free rate	-1.898*	0.151	-0.321	3.004***	-0.417	2.422***	0.345	0.935***
Option price	0.0253***	1.396***	0.0337***	0.0978***	0.0711***	1.686***	0.0902***	0.0591***
Days to maturity	-1.090***	-0.801***	-0.315*	-0.476***	-0.520***	-0.709***	-0.186***	0.00282
Days to maturity ^2	0.00934***	0.00728***	0.00308	0.00300	0.00472**	0.00591***	0.00152**	-0.000704
Calls / Puts	-2.269***	1.352***	1.406***	0.971***	0.327*	1.528***	-0.0412	0.0397
Year 90	9.374*		3.174	-23.92***		-19.31***	-1.735	-3.446**
Year 91	10.29***		1.186	-20.90***		-5.731***	-1.079	-1.879*
Year 92	3.124*		4.638**	-17.74***		2.882*	1.090**	0.305
Year 93	8.661***		1.876	-21.63***		-4.521**	0.318	-0.529
Year 94	3.537		-0.263	-40.67***		6.333***	-1.471***	-2.756***
Year 95	16.79***		-0.795	-22.47***	-4.707**	-3.421*	-0.648	-3.150***
Year 96	19.10***		-1.484	-18.90***	0.820	2.767*	-2.272***	-2.551***
Year 97	9.817***		0.133	-39.07***	-4.045**	7.344***	-2.779***	-1.537*
Year 98	15.62***	-4.247**	-3.849	-24.38***	-7.760***	-4.135**	-2.662***	-4.447***
Year 99	-0.390	-22.39***	-0.204	-18.72***	9.882***	-4.039	-0.814	-1.689**
Year 00	11.56***	-0.160	8.169***	-23.96***	-2.968*	-2.305	0.0153	-1.281
Year 02	-2.045	-0.938	4.040*	-3.475	-4.065**	3.920	-0.962	1.772
Year 03	-14.43***	-0.935	2.276	-5.186	-4.018*	7.515***	0.878	2.162*
Year 04	-11.03***	-5.281***	-4.970	-5.259	-4.963**	12.77***	1.835*	0.322
Year 05	-1.093	-3.686***	-1.263	-14.24***	-3.963***	13.67***	0.664	1.851**
Year 06	11.35***	-1.554	-8.555***	-14.04***	-16.10***	-2.968	-1.623***	-0.245
Year 07	10.23***	-4.810***	-9.432***	-13.93***	-3.193*	4.383*	-0.175	1.206
Year 08		-2.293	-5.076*	-11.07***	-14.63***	1.110	-0.241	0.140
Intercept	28.82***	24.18***	5.113	18.10***	15.46***	-0.466	3.985***	-3.533*
Obs.	4,800	24,044	24,415	28,857	6,635	17,188	18,132	16,317
R^2	0.784	0.626	0.449	0.536	0.769	0.617	0.705	0.637

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

Table 13. Regression Result: Precious Metal & Energy Commodities

$$S_{t,T} = \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt} + \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \mathrm{K} + \theta_{19} Y_{08} + \varepsilon_{t,T}$$

	Precious Metal			Energy			
Variables	Gold	Copper High Grade/ Scrap No. 2 Wire	Platinum	Soybean Oil/ Crude	Crude Oil	Natural Gas	Heating Oil #2
Open Interest	0.193	-2.213	-5.133	-3.130***	-0.960	-5.900***	3.316*
Log(s/k)	-10.49***	2.439***	-7.823***	-7.919***	2.778***	-13.65***	-5.614***
Log(s/k)^2	172.6***	97.28***	196.9***	135.4***	64.16***	43.27***	45.84***
Historical volatility (20 days)	0.120**	0.143***	0.316***	0.105**	0.0790**	0.232***	0.241***
Historical return (20 days)	15.14	19.78	-9.863	22.83***	27.16***	-0.365	11.59*
Risk free rate	0.927*	2.703**	0.976	3.102***	0.469	-0.0325	0.656
Option price	0.00650***	0.0366***	0.0359***	0.377***	-0.0236	-0.685***	2.444***
Days to maturity	-0.443***	-0.436***	-0.219	-0.266***	-0.274**	-0.250*	0.0374
Days to maturity ^2	0.00399***	0.00394**	0.00181	0.00197*	0.00211	0.00236	0.0000814
Calls / Puts	0.281***	0.228*	0.0494	0.392***	0.582***	-0.567***	1.259***
Year 90	-8.777***	-12.61***		-12.92***	-15.05***		-5.697*
Year 91	-2.100	-2.403	-5.269***	-8.180***	-0.574		5.911*
Year 92	-0.479	2.402**	0.612	4.147***	2.795**	13.35***	5.495***
Year 93	-3.336**	2.416	-2.275	1.732	2.141	15.77***	7.509***
Year 94	0.179	-2.020*	2.840	-0.762	0.224	13.30***	2.545
Year 95	-1.106	-3.526**	0.584	-3.524**	4.038***	10.66***	7.974***
Year 96	0.598	-5.221***	3.529***	-2.537**	-2.564**	7.212***	1.591
Year 97	-4.550***	-3.691***	-5.328**	-0.604	0.381	5.628	4.178**
Year 98	-4.677***	-2.293**	-1.657	0.175	-2.112*	16.38***	0.721
Year 99	-1.662*	-2.639***	3.038**	-1.279	0.447	14.92***	1.610
Year 00	0.544	-1.410	-8.763***	0.397	-4.168***	14.80***	-3.485*
Year 02	1.657	5.596*	1.837	5.567***	5.345***	12.35***	5.776**
Year 03	-1.486	5.225	3.871	8.203***	5.127**	13.34***	-0.405
Year 04	-1.995	-0.720	-2.390	4.641**	0.556	11.23***	-1.694
Year 05	-2.144*	4.597**	3.011	3.831**	-0.108	15.09***	-1.887
Year 06	-11.51***	-6.666***		1.800*	2.677**	10.43***	3.532*
Year 07	-7.887***	-8.318***		-0.0603	1.258	21.01***	2.520
Year 08	-9.961***	-8.120***		-6.426**	-7.845***	18.79***	-2.274
Intercept	10.56***	-0.0296	-2.433	-6.247*	5.442	-11.05*	-14.16***
Obs.	40,892	60,822	7,967	20,796	84,521	75,516	79,175
R^2	0.675	0.481	0.415	0.570	0.323	0.218	0.258

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

Table 14. Regression Result: All Markets

$$S_{t,T} = \alpha + \beta_1 Q_t + \beta_2 \log\left(\frac{F}{k}\right)_t + \beta_3 \log\left(\frac{F}{k}\right)_t^2 + \beta_4 \sigma_{-20}^2 + \beta_5 R_{-20} + \beta_6 r_f + \beta_7 P_t^{opt}$$

$$+ \beta_8 T + \beta_9 T^2 + \beta_{10} cp + \theta_1 Y_{90} + \theta_2 Y_{91} + \ldots + \theta_{19} Y_{08} + \sum_{m \in Exchanges} \lambda_m Dexch_m + \varepsilon_{t,T}$$

Variables	All options	Variables	All markets
Open Interest	-0.0215	Year 90	-3.369***
Log(s/k)	-8.353***	Year 91	0.582
Log(s/k)^2	95.60***	Year 92	2.319***
Historical volatility (20 days)	-0.0128	Year 93	1.085**
Historical return (20 days)	-0.174	Year 94	0.661
Risk free rate	0.572**	Year 95	0.623
Option price	0.0312***	Year 96	-0.830
Days to maturity	-0.303***	Year 97	-1.538**
Days to maturity ^2	0.00253***	Year 98	0.596
Calls / Puts	0.845***	Year 99	0.874
Chicago Mercantile Exchange	2.929***	Year 00	1.251*
Commodity Exchange	1.130***	Year 02	3.310***
Coffee, Sugar & Cocoa Exchange	2.761***	Year 03	2.011**
LIFFE Commodity Exchange	1.757***	Year 04	-0.205
Minneapolis Grain Exchange	-1.110***	Year 05	1.613***
New York Cotton Exchange	-0.916***	Year 06	-0.0988
New York Mercantile Exchange	-1.462***	Year 07	-0.128
Winnipeg Commodity Exchange	1.102***	Year 08	-3.458***
Intercept	5.965***		
Obs.	730,940		
R^2	0.307		

Note:
(a) t statistics in parenthesis
(b) * p< 0.05, ** p< 0.01, *** p< 0.0001

CHAPTER 5. CONCLUSION

Because of the well-known weaknesses of current option-pricing models, it would be naïve for option traders to quote option prices based purely on an options-pricing model. Among all option valuation models, the Black and Scholes model (BSM) is still the foundation for pricing vanilla options. Practitioners use the BSM not only to price plain-vanilla options, but also to give estimates of other more sophisticated options, and to invert the model to determine the implied volatility that can be used for the calibration of other valuation models. The imperfections of the BSM are manifest in the form of bias of implied volatility as an estimator of future-realized volatility, and as the volatility smile. Over the past 30 years, researchers have been struggling to explain the source of such smiles. Although several more complicated valuation models have been developed and volatility smiles can be produced, these models often do not have closed form solutions and are computationally intensive. Moreover, several studies have found that complexity costs of the more advanced models usually outweigh the drawbacks of the simpler BSM.

Among practitioners, the BSM remains the most popular option valuation model due to three major factors. First, more sophisticated models are computationally intensive, making them virtually impossible for option traders to use in their daily trading activities. Second, the calibration of the new models is time-consuming and sometimes even impossible. Finally, practitioners still must rely on the implied volatility of plain vanilla options to calibrate parameters of the other option-valuation models.

Therefore, instead of imposing new assumptions on the underlying asset and the volatility structure in order to develop a new valuation model for options as done in most other studies, this research takes a different approach in which the bias, not the option price itself, is being modeled. The idea of modeling the bias stems from the fact that the BSM is still the most widely used model among practitioners. This means that the market place simply views the bias as a correction of imperfections resulting from the BSM. Hence, modeling the bias itself as opposed to developing a new valuation model seems to be a more useful approach for practitioners.

In this study, the volatility bias is determined from a partial equilibrium framework which contains economic insight into the causes of the bias. According to the proposed model, the goods that are being traded are the services of options writers who are protected by trading rules that require access to a seat on the exchange to avoid service fees associated with trading. This model allows us to replicate the actual option market mechanism.

Although it could be argued that some researchers have employed the ad-hoc BSM model to empirically explain the bias, however, to my knowledge this theoretical approach has never been described in the literature before. Model equilibrium results from the agent's utility maximization problem in which the option writer is assumed to have monopoly power in order to reflect the advantage and skill that option writers possess. In order to obtain a certain level of bias in terms of dollars, the monopolist has to inflate the implied volatility for options. The two scenarios considered here produce a downward sloping volatility and a volatility smile. A combined mixture of these two scenarios should provide different curvatures for the volatility curve. Moreover, by changing the time to maturity of the options, different shapes of the volatility term structures can be reproduced. As a result, this model not only provides economic intuition behind the bias, it is also the first model that can explain both the volatility smile and the volatility term structure.

Discussion of empirical results was divided into two parts: (i) to show the evidence of bias and (ii) to show determinants of the bias. For ATM options, in nineteen out of twenty-six markets, the implied volatility is an upward-biased estimator of the realized volatility. The difference between this study and the previous literature is that the previous literature focused on the existence of a bias where in this study, the proposed model predicts a positive bias which is what we find in the data. Although the implied volatility appears to be an unbiased-biased estimator for the realized volatility in the cotton, oats, wheat No. 2, cocoa, orange juice, and heating oil markets for ATM options, however, for non-ATM options, implied volatility appears to be an upward-biased estimator for the realized volatility in all markets. Therefore, in almost all cases, we fail to reject the hypothesis that the implied volatility is a positive biased estimator of future realized volatility.

The theoretical model also suggests that the implied volatility's bias behavior is caused by the quantity hedged, the strike, volatility, futures price, the risk-free rate, option prices, and days to maturity. The second section of the empirical findings presents results using these variables to test that the bias is non-constant across strikes, times to maturity, puts and calls, option year, and exchanges. The empirical results fail to reject all of these hypotheses. Hence, the bias varies according to these variables. Additionally, the result also shows that the Chicago Mercantile Exchange has the highest bias among all the Exchanges. The introduction of electronic trading does not seem to have had any influence of the size or the presence of a bias.

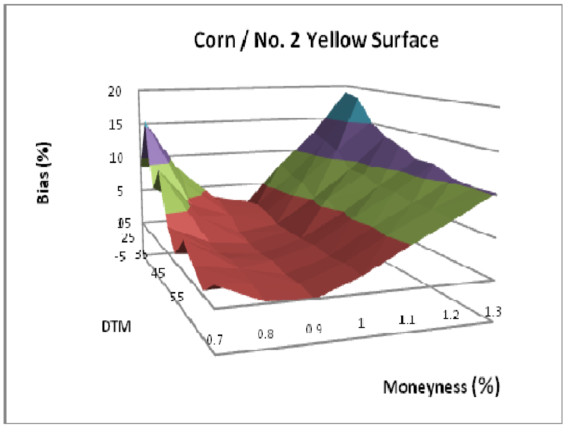
In most markets, the *Open Interest* and the *Historical return (20days)* variables do not appear to have much impact. However, the *Historical volatility (20days),* the Risk free rate, and the Option price variables are shown to have a positive impact on the bias in most markets. Finally, the empirical model appears to explain the bias reasonably well with 30%-40% R^2 in eleven markets and more than 50% R^2 in thirteen markets.

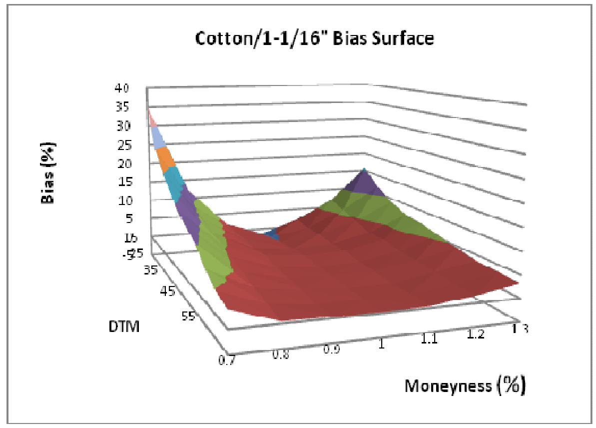
Future research could adapt this model to conduct hedging strategies and to compare hedging performance between this model and other existing models. Moreover, since this model not only suggests that, in most markets, the implied volatility is a biased estimation of the realized volatility, variables possibly explaining the bias are also suggested, hence, future research should be done looking at employing this model as another tool for forecasting future volatility.

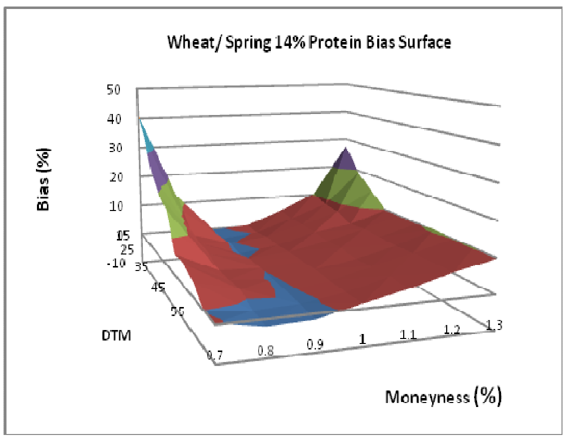
Finally, the results shown here also shave a practical application for those who would like to use options prices to find the markets estimate of implied volatility. To do this one would first subtract the average bias presented here (or the average transaction fee for a round trip option purchase) from the actual option premium before solving for the implied volatility for an at the money option.

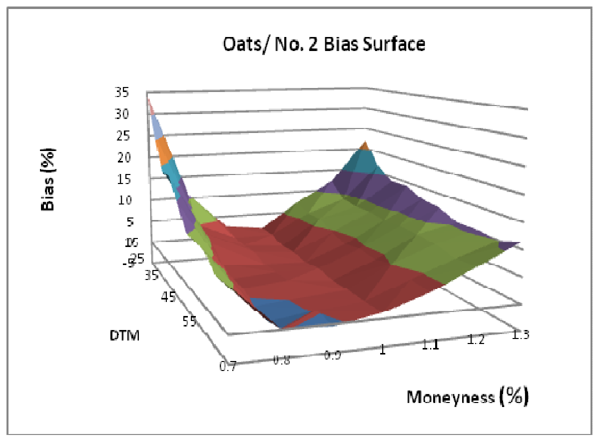
APPENDIX A. THE BIAS SURFACE BY MARKETS

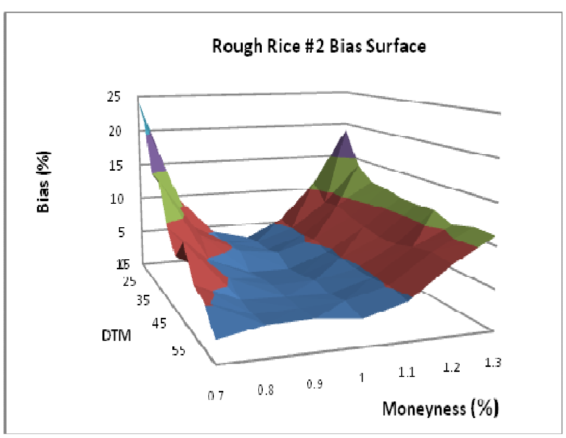
Agricultural Commodities

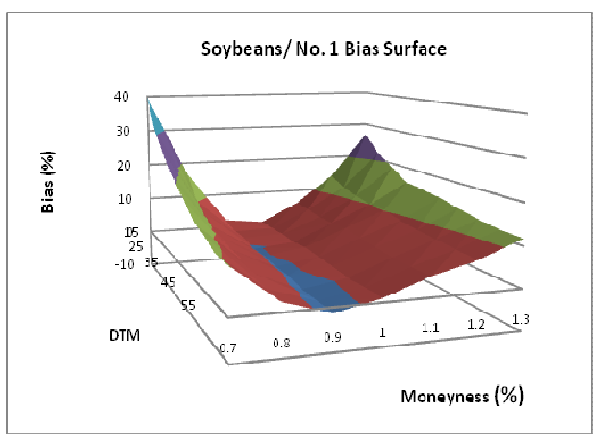


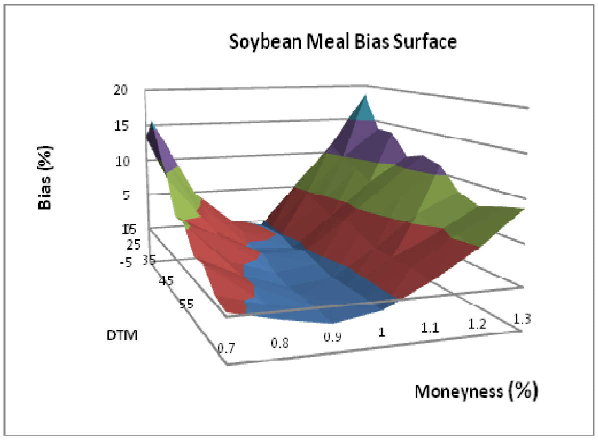


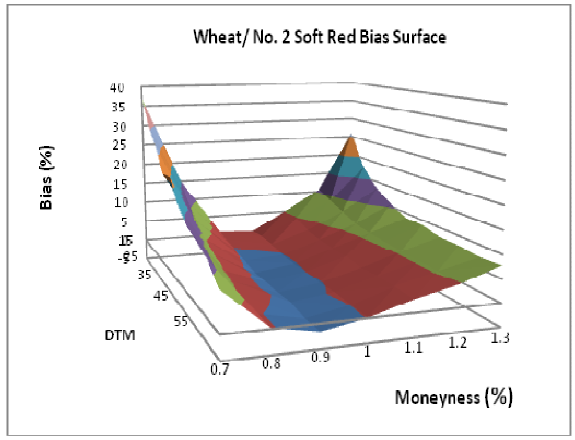


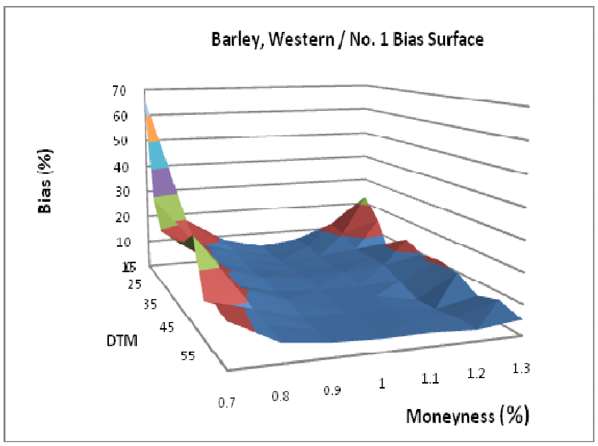


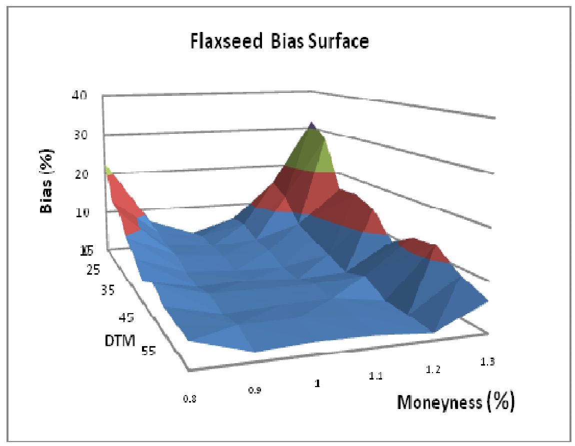


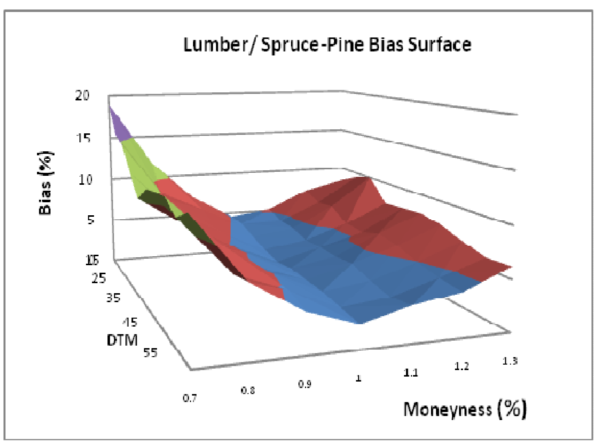



Soybean Meal Bias Surface
Bias (%)
DTM
Moneyness (%)

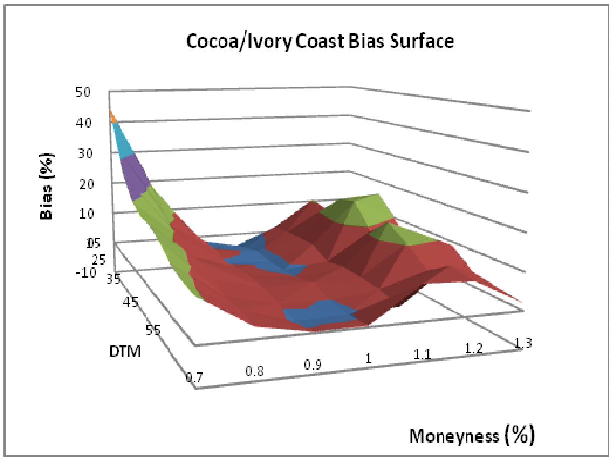

Wheat/ No. 2 Soft Red Bias Surface
Bias (%)
DTM
Moneyness (%)

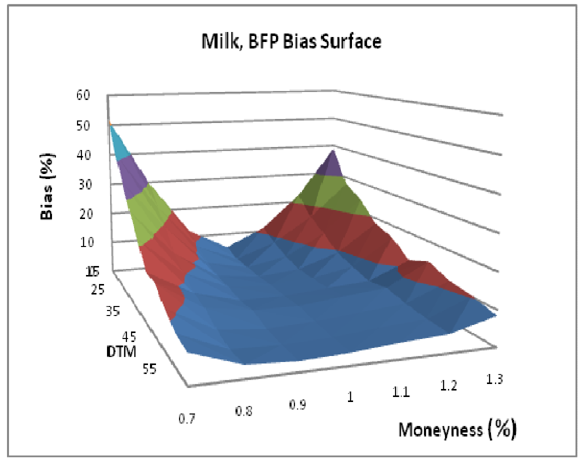

Barley, Western / No. 1 Bias Surface
Bias (%)
DTM
Moneyness (%)

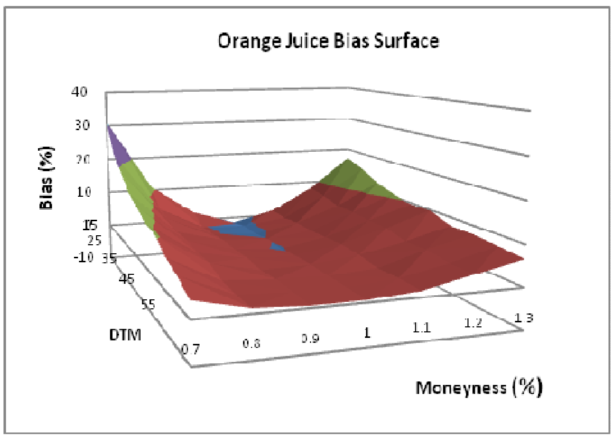

Flaxseed Bias Surface
Bias (%)
DTM
Moneyness (%)

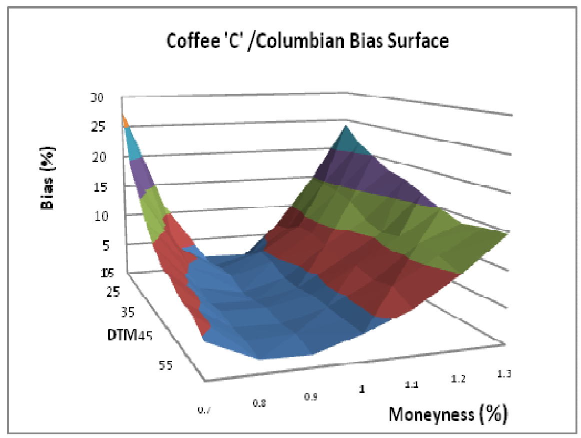

Lumber/ Spruce-Pine Bias Surface
Bias (%)
DTM
Moneyness (%)

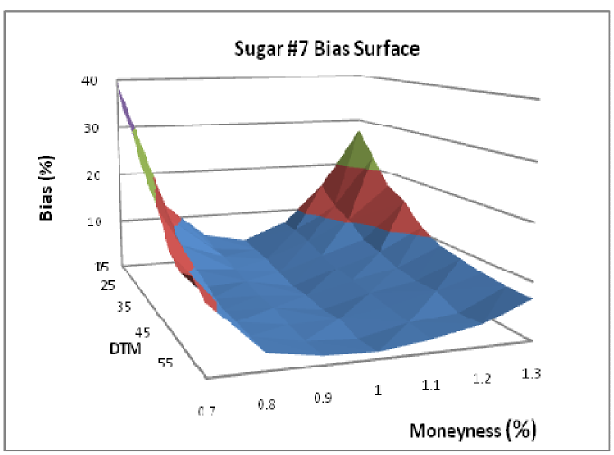
Soft Commodities

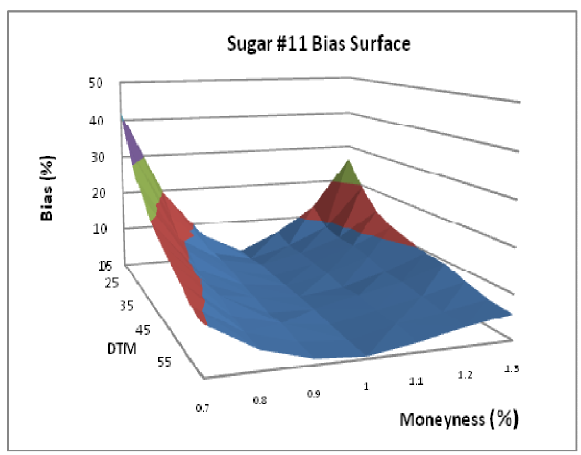




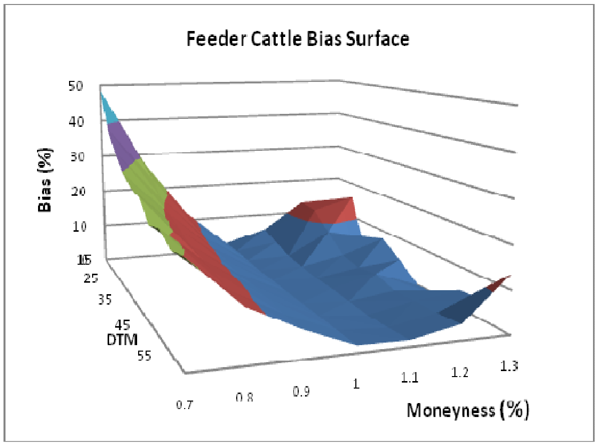


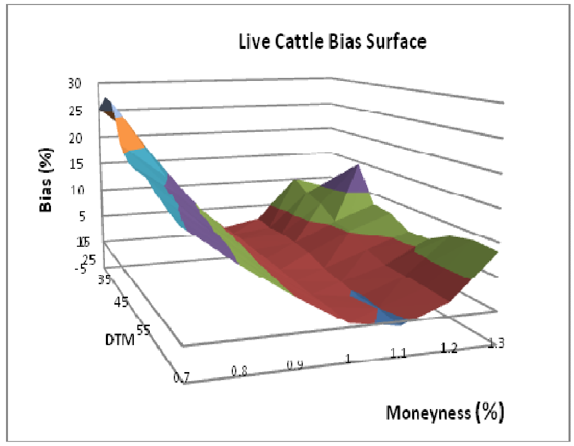




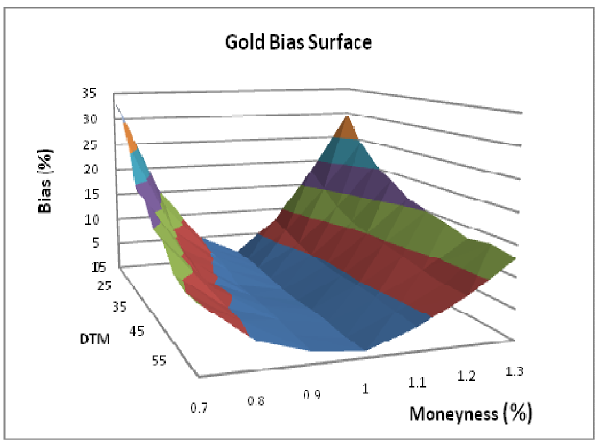


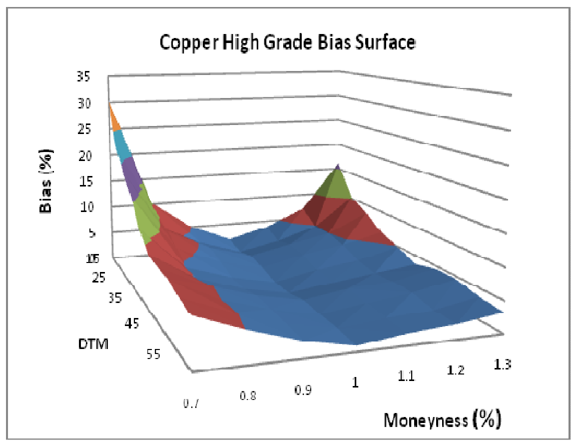
Livestock Commodities

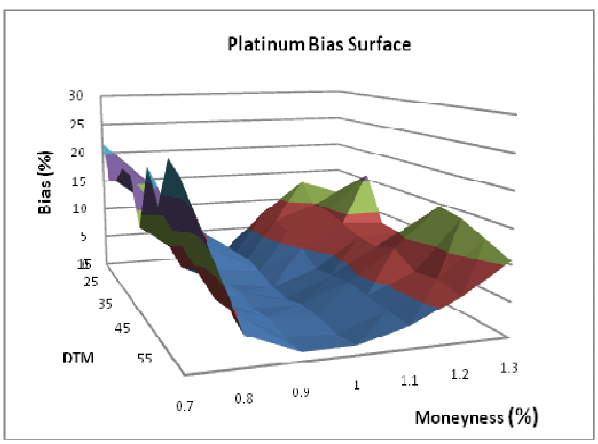




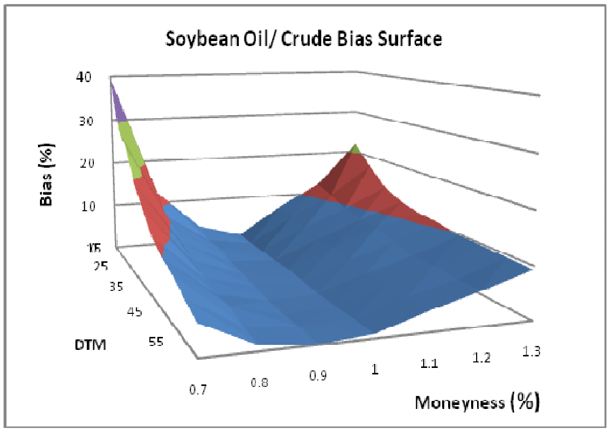
Precious Metal Commodities

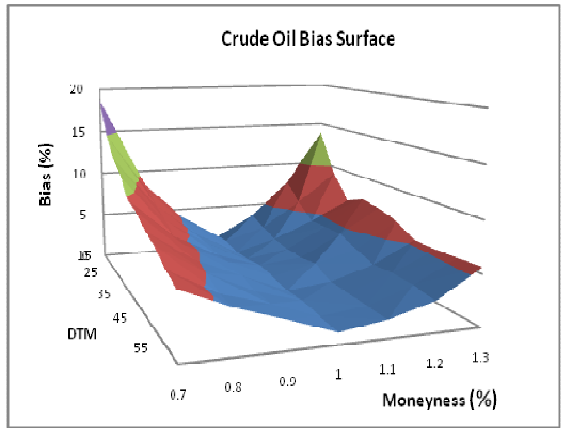


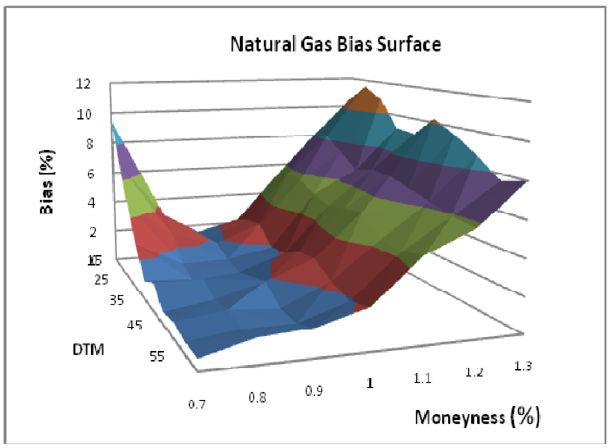


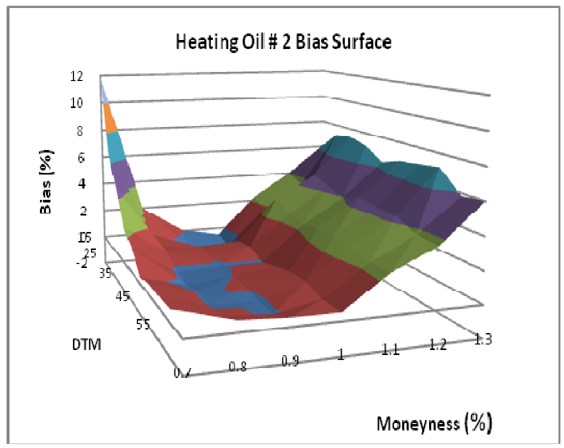


Energy Commodities









APPENDIX B. MATLAB CODE FOR NUMERICAL ANALYSIS

Main Program

```
A0 = [1;1;1;1;1]
lb = [0;0;0;0;0]
ub = [10;10;10;10;10]
% change ub according to Q

[A , fval] = fmincon(@MonopolistProgram, A0 ,[],[], [], [], lb, ub,
@MonopolistUnConst)

[A , fval] = fmincon(@MonopolistProgram, A0 ,[],[], [], [], lb, ub,
@MonopolistConst1)
```

function [profitSum] = MonopolistProgram(A)

```
global H
global c
global a
global f0
global mu
global sigma
global rf
global Q
global k
global T

H = 1;
c = 0.1;

a = 1;
f0 = 1;
mu = 1;
sigma = 0.25;
T = 0.5;
rf = 0.05;
Q = 10;

k = [.8; .9 ; 1 ; 1.1 ; 1.2];

profit = 0;
for i = 1:1: 5

      % part 2) utility at time 1
      p0 = exp(-rf*T)*f0;
      d1 = ( log(p0/k(i,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
      d2 = d1 - sigma * sqrt(T);
      PoptPut = exp(-rf*T)* ( k(i,1) *  normcdf(-d2) - f0 * normcdf(-d1)
      );
```

```
        [p1,interval] = qnwtrap(30000, 0.00000001, 200);
        sigmaLog = sqrt( T ) * sigma;
        muLog = T*rf - T*(sigma^2)/2;

        piece1 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(i,1) -
        p1,0).* exp( (-a)*( A(i,1).*max(k(i,1)-p1,0) + Q.*p1) - (1/2).*
        ((log(p1) - log(p0)-muLog)./sigmaLog).^2));
        piece2 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
        A(i,1).*max(k(i,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
        muLog)./sigmaLog).^2));

        rev = (piece1/piece2)*(1/(1+rf)) ;

        futureRev =  interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*min(p1 -
        k(i,1) ,0)*A(i,1).* exp(- (1/2).* ((log(p1) - log(p0)-
        muLog)./sigmaLog).^2));

        profitIndividual = rev*A(i,1)  - c*A(i,1) - futureRev;

        profit = profit + profitIndividual;

end

profitSum = - (profit - H)
```

function [const1, czero] = MonopolistUnConst(A)

```
global H
global c
global a
global f0
global mu
global sigma
global rf
global Q
global k
global T

% ======= Set 1 ============================
% % % k = [.95; 1; 1.05];
p0 = exp(-rf*T)*f0;
d11 = ( log(p0/k(1,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d12 = ( log(p0/k(2,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d21 = d11 - sigma * sqrt(T);
d22 = d12 - sigma * sqrt(T);
PoptPut1 = exp(-rf*T)* ( k(1,1) *  normcdf(-d21) - f0 * normcdf(-d11) );
PoptPut2 = exp(-rf*T)* ( k(2,1) *  normcdf(-d22) - f0 * normcdf(-d12) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;
```

```
piece1 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(1,1) -
p1,0).* exp( (-a)*( A(1,1).*max(k(1,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece2 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(1,1).*max(k(1,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S1 = (piece1/piece2)*(1/(1+rf)) - PoptPut1;

piece11 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(2,1) -
p1,0).* exp( (-a)*( A(2,1).*max(k(2,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece22 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(2,1).*max(k(2,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S2 = (piece11/piece22)*(1/(1+rf)) - PoptPut2;


% ======= Set 3 ============================
p0 = exp(-rf*T)*f0;
d13 = ( log(p0/k(3,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d23 = d13 - sigma * sqrt(T);
PoptPut3 = exp(-rf*T)* ( k(3,1) *  normcdf(-d23) - f0 * normcdf(-d13) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(3,1) -
p1,0).* exp( (-a)*( A(3,1).*max(k(3,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(3,1).*max(k(3,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S3 = (piece3/piece4)*(1/(1+rf)) - PoptPut3;


% ======= Set 4 ============================
p0 = exp(-rf*T)*f0;
d14 = ( log(p0/k(4,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d24 = d14 - sigma * sqrt(T);
PoptPut4 = exp(-rf*T)* ( k(4,1) *  normcdf(-d24) - f0 * normcdf(-d14) );



[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(4,1) -
p1,0).* exp( (-a)*( A(4,1).*max(k(4,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
```

```
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(4,1).*max(k(4,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S4 = (piece3/piece4)*(1/(1+rf)) - PoptPut4;


% ======= Set 5 ============================
p0 = exp(-rf*T)*f0;
d15 = ( log(p0/k(5,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d25 = d15 - sigma * sqrt(T);
PoptPut5 = exp(-rf*T)* ( k(5,1) *  normcdf(-d25) - f0 * normcdf(-d15) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(5,1) -
p1,0).* exp( (-a)*( A(5,1).*max(k(5,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(5,1).*max(k(5,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S5 = (piece3/piece4)*(1/(1+rf)) - PoptPut5;

% ==============================================================
PoptPut1
PoptPut2
PoptPut3
PoptPut4
PoptPut5

Sall = [S1; S2 ; S3; S4; S5 ]
const1 = [-S1 ;  -S2 ; -S3 ; -S4 ; -S5];
czero = [];
```

function [czero, const1] = MonopolistConst1(A)

```
global H
global c
global a
global f0
global mu
global sigma
global rf
global Q
global k
global T

% ======= Set 1 & 2 ============================
% % % k = [.95; 1; 1.05];
p0 = exp(-rf*T)*f0;
d11 = ( log(p0/k(1,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d12 = ( log(p0/k(2,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
```

```
d21 = d11 - sigma * sqrt(T);
d22 = d12 - sigma * sqrt(T);
PoptPut1 = exp(-rf*T)* ( k(1,1) *  normcdf(-d21) - f0 * normcdf(-d11) );
PoptPut2 = exp(-rf*T)* ( k(2,1) *  normcdf(-d22) - f0 * normcdf(-d12) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece1 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(1,1) -
p1,0).* exp( (-a)*( A(1,1).*max(k(1,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece2 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(1,1).*max(k(1,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S1 = (piece1/piece2)*(1/(1+rf)) - PoptPut1;

piece11 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(2,1) -
p1,0).* exp( (-a)*( A(2,1).*max(k(2,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece22 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(2,1).*max(k(2,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S2 = (piece11/piece22)*(1/(1+rf)) - PoptPut2;


% ======= Set 3 ============================
p0 = exp(-rf*T)*f0;
d13 = ( log(p0/k(3,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d23 = d13 - sigma * sqrt(T);
PoptPut3 = exp(-rf*T)* ( k(3,1) *  normcdf(-d23) - f0 * normcdf(-d13) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(3,1) -
p1,0).* exp( (-a)*( A(3,1).*max(k(3,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(3,1).*max(k(3,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S3 = (piece3/piece4)*(1/(1+rf)) - PoptPut3;


% ======= Set 4 ============================
p0 = exp(-rf*T)*f0;
d14 = ( log(p0/k(4,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d24 = d14 - sigma * sqrt(T);
PoptPut4 = exp(-rf*T)* ( k(4,1) *  normcdf(-d24) - f0 * normcdf(-d14) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
```

```
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(4,1) -
p1,0).* exp( (-a)*( A(4,1).*max(k(4,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(4,1).*max(k(4,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S4 = (piece3/piece4)*(1/(1+rf)) - PoptPut4;


% ======= Set 5 ============================
p0 = exp(-rf*T)*f0;
d15 = ( log(p0/k(5,1)) + T*(rf+sigma^2/2)  )  / ( sigma * sqrt(T) );
d25 = d15 - sigma * sqrt(T);
PoptPut5 = exp(-rf*T)* ( k(5,1) *  normcdf(-d25) - f0 * normcdf(-d15) );

[p1,interval] = qnwtrap(30000, 0.00000001, 200);
sigmaLog = sqrt( T ) * sigma;
muLog = T*rf - T*(sigma^2)/2;

piece3 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).*max(k(5,1) -
p1,0).* exp( (-a)*( A(5,1).*max(k(5,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1)
- log(p0)-muLog)./sigmaLog).^2));
piece4 = interval'*  ( (1./(sigmaLog*sqrt(2*pi).*p1)).* exp( (-a)*(
A(5,1).*max(k(5,1)-p1,0) + Q.*p1) - (1/2).* ((log(p1) - log(p0)-
muLog)./sigmaLog).^2));
S5 = (piece3/piece4)*(1/(1+rf)) - PoptPut5;

% ============================

PoptPut1
PoptPut2
PoptPut3
PoptPut4
PoptPut5

Sall = [S1; S2 ; S3; S4; S5 ]

const1 = [S1-S2; S1-S3; S1-S4; S1-S5; S2-S3; S2-S4; S2-S5; S3-S4; S3-S5;
S4-S5];

czero = [];
```

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